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Numerical prediction of disorder effects in solid foams using a probabilistic homogenization scheme

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ABSTRACT

The present study is concerned with a numerical prediction of uncertainties in the macroscopic mechanical properties of microheterogeneous materials with uncertain microstructure. As a model material, solid foams are employed. The stochastic information on the uncertainty is gained in multiple numerical homogenization analyses of small-scale testing volume elements. The local relative density, the cell size distribution, the cell geometry and the spatial orientation of the testing volume elements are assumed to form the set of the relevant stochastic variables. Selected microstructural cases are analyzed for their macroscopic material response. Based on the probability distributions for the stochastic variables defining the microstructures of the testing volume elements, the probability distributions for the mesoscopic material properties are obtained. For the numerical homogenization of the testing volume elements, an enhanced finite element technique is employed, where the components of the macroscopic deformation gradient are introduced as generalized degrees of freedom. Assuming periodic boundary conditions, the global degrees of freedom interact with the conventional displacement degrees of freedom of the discretized microstructure via special boundary coupling elements. The mesoscopic stresses are obtained in a rather efficient manner as the generalized reaction forces for the global degrees of freedom.

1. Introduction

Solid foams made from polymeric or metallic materials are important materials in a variety of structural applications. Their main advantage is their rather low specific weight due to their high void volume fraction. Hence, solid foams are a natural choice in many fields of lightweight construction, such as cores for sandwich plates and shells. Compared to classical two-dimensional honeycomb cores, foams have the advantage that they can easily processed to any desired shape. Another advantage of solid foams is their high compressibility together with the fact that the compression occurs at an approximately constant effective stress level over a wide range. Therefore, solid foams are frequently used in packaging technology, for personal protection systems such as safety helmets and other types of impact protection systems. Furthermore, solid foams have the ability to take over additional functions such as acoustic or thermal insulation in multifunctional devices.

The design and numerical analysis of structures and components consisting partially or fully of foamed materials is preferably be performed in terms of macroscopic "effective" properties rather than by means of a detailed consideration of their real microstructure. The effective material response can be determined experimentally or numerically. Although the experimental characterization cannot be replaced entirely by numerical methods, numerical analyses complementing the experimental characterization may serve to reduce the experimental

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effort significantly. The numerical analysis may help to fill in gaps in experimental data bases, enables parameter studies and a rigorous algorithmic optimization of the material. Furthermore, it may provide a deeper insight into the underlying microstructural mechanisms of deformation and thus a deeper understanding of the material behavior.

The pioneering study on the numerical prediction of the macroscopic material response of foams has been published by Gent and Thomas (1963). Since then, a considerable body of literature has been provided. Most of the early studies are concerned with the analysis of idealized regular microstructural models such as Kelvin's (Thomson, 1887) tetrakaidecahedral foam model (e.g. Warren and Kraynik, 1997), Christensen's (1987) pentagonal dodecahedron cell model or the rectangular cell model employed by Gibson and Ashby (1982).

Regular models can successfully be employed for determination of the mean effective properties of foams. However, one of the main disadvantages of solid foams is their random disordered microstructure. Especially in conjunction with large cell sizes as in the case of most metallic foams, the microstructural disorder may result in a distinct scatter and uncertainty in their macroscopic properties. For the variability of the Young's modulus of ALPORAS aluminum foam, Ramamurty and Paul (2004) report standard deviations of up to 16.7% of the corresponding mean value and total scatter band widths of up to 69% of the mean Young's modulus. Similar results were obtained by Despois et al. (2006) in a screening analysis on the effective Young's modulus and yield stress of different metal foams. Hence, the reproducibility of the effective properties of foams is in many cases limited. Although the variability in the macroscopic material response decreases with increasing sample size due to self-averaging effects (Zhu et al., 2000), the experimental results by Ramamurty and Paul (2004) and Despois et al. (2006) as well as numerical studies by van der Burg et al. (1997) or Kanaun and Tkachenko (2006) reveal that rather large sample sizes consisting of 1000 or more cells might by necessary in order to obtain stable results.

The uncertainty in the macroscopic properties caused by the disordered microstructure requires the application of stochastic methods. Stochastic analysis techniques enable an assessment of the macroscopic material uncertainty in terms of the probability distributions for the macroscopic properties. The first stochastic approaches used in foam mechanics were directed to the problem of random orientation of the cell models in space (e.g. Hall, 1993). Fortes and Ashby (1999) were concerned with a randomly oriented single strut of an open cell foam whereas Warren and Kraynik (1997) used the variational Hashin–Shtrikman approach to account for randomness of the spatial orientation of their Kelvin foam model. For the same purpose, Cuitiño and Zheng (2003) employed a Taylor averaging procedure.

Further uncertainties are due to the irregular random shape of the cells. In order to account for these effects, more recent studies frequently use irregular random microstructures generated either by a Voronoï (1908) process or by perturbation techniques based on an regular base microstructure. Whereas van der Burg et al. (1997) employed a large scale representative volume element for determination of the mean effective moduli of the material, repeated analyses of small or medium scale volume elements were employed in order to assess the scatter in terms of the standard deviation of the macroscopic material properties, among others, by Zhu et al. (2000) or one of the present authors (Hohe and Becker, 2005).

In the present study, a similar strategy is employed. In contrast to the previous direct Monte-Carlo approaches, where the microstructure of the individual "testing volume elements" was generated randomly throughout the permitted range of the stochastic variables, a weighted procedure is employed. In this context, a limited number of numerical experiments is performed with pre-defined values of the stochastic variables. Similar as proposed by Schraad and Harlow (2006), the results of the individual analyses are provided with the individual probabilities for the occurrence of the underlying microstructures. From the results, the probability distributions for the mesoscopic material properties are obtained. For determination of the mesoscopic properties, a strain energy based homogenization procedure (Hohe and Becker, 2005) is adopted. This scheme defines the equivalence of the given microstructure and the quasi-homogeneous effective medium by the condition that an equivalent deformation of the microstructure and the effective medium has to result in equal strain energy density. Within the present study, the approach is re-formulated for a finite element implementation by application of the strain energy based equivalence condition to the virtual work on micro and macro level. Together with the implementation of special boundary coupling elements, a finite element formulation is obtained, where the components of the mesoscopic deformation gradient enter as direct degrees of freedom.

2. Probabilistic homogenization scheme

2.1. Effective material properties

The general static homogenization problem of the definition of macroscopic or "effective" properties for a microheterogeneous material can be defined as follows: Consider a mechanical body Ω with characteristic dimension L according to Fig. 1. The body is bounded by external boundaries $\partial\Omega^{u}$ with prescribed displacements $u_i = u_i^0$ and $\partial\Omega^{t}$ with prescribed tractions $t_i = t_i^0 = \sigma_{ij} n_j$. Here, σ_{ij} are the components of the Cauchy stress tensor whereas n_j are the components of the outward normal unit vector. The body Ω is assumed to consist of a microheterogeneous medium.

The numerical analysis of the structural response of the body Ω based on a detailed microstructural model would be numerically inefficient and in many cases merely impossible due to the extensive number of degrees of freedom involved. To circumvent these expenses, the body Ω is substituted by a similar body Ω^* with similar external shape and dimension L, subject to similar boundary and loading conditions $u_i^* = u_i^{0*}$ on $\partial\Omega^{u*}$, $t_i^* = t_i^{0*}$ on $\partial\Omega^{t*}$ respectively. In contrast to the original body Ω, the substitute