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Closed-form expressions for the macroscopic flexural rigidity coefficients of periodic brickwork

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ABSTRACT

Approximate expressions for the macroscopic out-of-plane elastic coefficients of brick masonry with a regular pattern are derived in closed form using a homogenization approach for periodic media. Following an approach similar to the Method of Cells for fiber reinforced composites, a (piecewise-)differentiable expression depending on very a limited number of degrees of freedom and fulfilling suitable periodicity conditions is proposed for the microscopic transverse displacement field over any Representative Volume Element (RVE). Some of the equilibrium conditions at the interfaces between bricks and mortar joints are also fulfilled. By averaging the moment and curvature fields over the RVE, the macroscopic bending stiffness coefficients can be explicitly obtained. Using the FE solution of a masonry panel subjected to elementary load conditions as a benchmark, the proposed approach is found to accurately match the numerically obtained stiffness coefficients, for masonry elements of different geometry and different mechanical properties. In several instances, the proposed expressions agree with the numerical predictions better than other analytical expressions available in the literature.

1. Introduction

Masonry walls are basically conceived to undergo vertical compressive loads. This notwithstanding, in several instances they can be subjected also to bending and twisting moments. Typically, this occurs when the wall is acted upon by transverse loads, as a consequence e.g. of seismic actions. Also, if the vertical loads are not perfectly centered, the wall experiences moments in addition to in-plane forces.

Several authors have addressed the study of the mechanical behavior of masonry walls subjected to out-of-plane actions. Estimates for the macroscopic flexural stiffness coefficients of brickwork were presented in [1] using a homogenization approach for periodic media: mortar joints are assumed to be interfaces of vanishing thickness, and units to be either rigid or elastic. Whereas the bending stiffness coefficients are simply given by the homogenized in-plane coefficients computed in plane strain conditions times $t^3/12$ (t being the wall thickness), upper and lower bounds for the torsional stiffness are formulated. The accuracy of the theoretical estimates was assessed in [2] using the Finite Element Method to analyze the 3D model of a Representative Volume Element (RVE). A similar numerical approach was followed in [3], where the macroscopic moments and curvatures experienced by the wall are explicitly related to the displacements and the forces at some 'master nodes' of the RVE.

More attention has been devoted to the prediction of the ultimate strength properties of masonry walls under transverse loads, which are responsible for the collapse of many historic structures in seismic zone. This was done e.g. in [4,5], applying limit analysis within the framework of homogenization theory for periodic media and deriving macroscopic strength surfaces in the space of the bending and twisting moments.

In this paper, an approach recently proposed to estimate the in-plane macroscopic elastic coefficients of masonry [6] is extended to predict the macroscopic bending and twisting stiffness coefficients of masonry walls with regular brick pattern. These parameters are of interest in the assessment of masonry structures under service loads, as required by codes of practice. The proposed approach stems from the so-called Method of Cells (MoC), originally proposed by Aboudi [7] to predict the macroscopic mechanical behavior of periodic unidirectional fiber-reinforced composites. Unlike the approach followed in [1], the MoC allows the finite thickness of the joints to be explicitly taken into account.

The layout of the paper is as follows. First, in Section 2 the fundamentals of homogenization theory for periodic media are briefly recalled, with special emphasis on heterogeneous 2D solids experiencing moments and curvatures. In Section 3 the original approach proposed to derive the macroscopic bending (Section 3.1)

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and twisting (Section 3.2) stiffness coefficients of masonry is illustrated. In Section 4 a parametric study is carried out to estimate the macroscopic flexural stiffness of masonry walls with bricks and joints of different sizes and mechanical properties. The theoretical predictions are compared with the numerical results of FE models taken as benchmarks and with other analytical expressions available in the literature [1]. Finally, in Section 5 the main findings of the work are summarized and possible future developments are outlined.

2. Homogenization for periodic media

The fundamental concepts of homogenization theory for periodic media are briefly recalled hereafter. Readers are referred e.g. to [8] for more details.

Consider a 2D heterogeneous body in the plane (x_1, x_2), with mechanical properties periodically varying in space. The period must be sufficiently smaller than the size of the body, so that the real medium can be effectively replaced by a 'homogenized' one. The global (or macroscopic) properties of the homogenized medium are derived through the analysis of a Representative Volume Element (RVE), also called 'unit cell' and denoted by V hereafter.

Assume that the 2D body represents the mid-plane of a heterogeneous plate undergoing transverse loads and obeying Kirchhoff–Love theory for thin plates: the validity of this assumption for masonry walls will be numerically assessed in Section 4. Also, assume that (x_1, x_2) is a plane of elastic symmetry for the plate. Accordingly, the RVE experiences only transverse displacements, $w(x_1, x_2)$, whereas in-plane displacements can be neglected. Macroscopically, the RVE experiences bending curvatures, X_{11} and X_{22}, and a twisting curvature X_{12} (= X_{21}). According to [3], the microscopic displacement field over the RVE must be of the form

$$w = w_0 + (\boldsymbol{\omega} \wedge \mathbf{x})_3 + \frac{1}{2} X_{\alpha\beta} x_\alpha x_\beta + \tilde{w}(x_1, x_2), \tag{1}$$

where $\mathbf{x}$ is any point in the RVE, w_0 and $\boldsymbol{\omega}$ define any infinitesimal rigid body motion of the RVE, $\tilde{w}$ is periodic over V and summation over $\alpha, \beta = 1, 2$ is understood. The macroscopic moments acting on the RVE, $\mathbf{M}$, and the macroscopic curvatures, $\mathbf{X}$, are defined as [3]

$$\mathbf{M} = \frac{1}{|V|}\int_V \mathbf{m}(\mathbf{x}) dV \equiv -\frac{1}{|V|}\int_V \boldsymbol{\sigma}(\mathbf{x}) x_3 dV, \tag{2}$$

$$\mathbf{X} = \frac{1}{|V|}\int_V \boldsymbol{\chi}(\mathbf{x}) dV \equiv \frac{1}{|V|}\int_V \nabla\nabla w(\mathbf{x}) dV, \tag{3}$$

where $\boldsymbol{\sigma}(\mathbf{x})$ is the microscopic stress field, $\mathbf{m}(\mathbf{x})$ and $\boldsymbol{\chi}(\mathbf{x})$ are the microscopic moment and curvature fields, and ∇ is the in-plane gradient operator.

In linear elasticity, the macroscopic constitutive law can be written as

$$\mathbf{M} = \mathbf{D}^{hom} : \mathbf{X}, \tag{4}$$

where $\mathbf{D}^{hom}$ denotes the macroscopic bending stiffness tensor. Assuming masonry to be macroscopically orthotropic, D^{hom} has four independent components at most.

3. A Method of Cells-type approach for the determination of the macroscopic elastic out-of-plane coefficients of masonry

The Method of Cells (MoC) was originally proposed by Aboudi [7] to derive the macroscopic properties of unidirectional composites, reinforced by a doubly-periodic array of long, parallel fibers (FRCs). In the elastic field, this method allows the macroscopic elastic constants of FRCs to be derived in closed form according to the mechanical properties of fibers and matrix. It can also be extended to the elasto-plastic and the viscoelastic field. The original MoC comprises the following steps: (*i*) any RVE is subdivided into four rectangular sub-cells, in which displacements are supposed to vary linearly; (*ii*) continuity of the average displacements and tractions across any interface between adjacent sub-cells is enforced; (*iii*) the composite constitutive equations are obtained expressing the stresses averaged over the RVE (i.e. the macroscopic stresses) in terms of averaged (or macroscopic) strains.

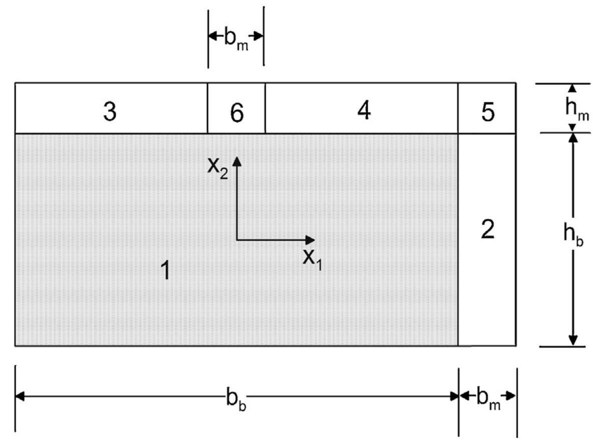

Fig. 1. Subdivision of any RVE into sub-cells.

A somehow similar approach was recently proposed in [6] to derive the macroscopic in-plane elastic and creep coefficients of brick masonry in closed form. The microscopic displacement field employed in [6] was later interpreted as collapse velocity field for the RVE, and approximate expressions for the macroscopic strength domain of masonry were derived according to a kinematic approach of limit analysis [10].

Except for stack bond masonry, because of the staggered position of the bricks from one course to another one, the RVE of periodic brickwork cannot be simply divided into four sub-cells as in the original MoC. For running bond or header bond brickwork, a possible choice of the RVE is shown in Fig. 1, along with its subdivision into six sub-cells. Sub-cell no. 1 corresponds to a brick; sub-cell no. 2 is a head joint; sub-cells 3 and 4 pertain to a bed joint; sub-cells 5 and 6 are intersections of bed and head joints (also called 'cross joints'). Note that the 2D RVE must be intended as the intersection of a 3D RVE, such as that considered in [3], with the mid-plane of a masonry wall.

From here onwards, x_1 denotes an axis parallel to the bed joints, x_2 an axis parallel to the head joints, and x_3 an axis parallel to the wall thickness (see Fig. 1); the origin of the coordinate system $Ox_1x_2x_3$ is placed in the mid-plane of the wall. b_b and h_b will denote the width and the height of the bricks, b_m the thickness of the head joints and h_m the thickness of the bed joints. The aspect ratio of the bricks will be denoted by $\alpha_B = b_b/h_b$. Other nondimensional geometric ratios that will be used in the continuation of the paper are $\alpha_b = b_m/b_b$ and $\alpha_h = h_m/h_b$. The wall thickness is assumed to be constant and will be denoted by t.

A curvature-periodic microscopic transverse displacement field, depending on a very limited number of parameters (or 'degrees of freedom', d.o.f.s) will be now proposed over the RVE. Following the steps of the MoC, the macroscopic out-of-plane stiffness coefficients of masonry (i.e. the Cartesian components of tensor $\mathbf{D}^{hom}$) will be estimated.

Assume bricks and mortar to be linear elastic isotropic materials and denote by E_b, ν_b (respectively, by E_m, ν_m) the Young's modulus and the Poisson's ratio of the brick (resp., of mortar).