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Spline interpolation with optimal frequency spectrum for vibration avoidance

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ABSTRACT

A major source of inaccuracy in CNC machines is unwanted vibrations induced by the frequency spectra of reference motion trajectory. This paper presents a novel approach where instead of filtering techniques, axis motion commands are generated with optimal frequency spectra in the first place. Tangential feedrate profile is defined as parametric spline, and its frequency spectrum is optimized with respect to structural dynamics of the machine. The optimization problem is solved efficiently using Quadratic Programming. Experimental results confirm that proposed technique can greatly improve surface finish during machining spline tool-paths without sacrificing from cycle time and contouring performance.

1. Introduction

Reference trajectory generation controls achievable accuracy and productivity of modern CNC machines in two major ways [1]. Firstly, reference trajectories must be planned within kinematic, e.g. velocity, acceleration and jerk, limits of the machine so that drives are not saturated, and a time-optimal motion is generated for high productivity [2–4]. Secondly, frequency spectrum of motion commands may excite lightly damped modes of the machine and induce unwanted vibrations. This can significantly deteriorate dynamic accuracy of machines [5,6]. As a result, the overreaching goal of modern trajectory generation algorithms has been to generate time-optimal motion commands with desired frequency spectra so that both productivity and high dynamic accuracy could be achieved.

There are two ways to generate time-optimal reference trajectories with desired frequency spectra. The most widely used approach is “pre-filtering”. Trajectories are initially time-optimized along a given path w.r.t. machine limits [2–4,10] and then frequencies that may excite natural modes of the machine are removed by pre-filtering. Notch filters, input shapers [5,6], or moving average filters [7,8] are commonly used. Although pre-filtering is effective in avoiding unwanted vibrations, it introduces motion delay and elongates cycle times. More importantly, filter dynamics disturb multi-axis motion synchronization, which causes large contouring errors, and needs to be compensated [6–8].

The second approach is to generate trajectories with smooth kinematic profiles in the first place so that their frequency spectrum is well attenuated, and filtering is not needed. Bell-shaped [9,10] acceleration profiles are favored in practice. But, minimum-jerk trajectories are known to be the most effective [2,3]. Jerk is the derivative of acceleration and acts as a high-pass filter in frequency domain. Therefore, minimizing jerk attenuates high frequency spectrum of acceleration. However, there is no direct mapping between the jerk content, or jerk limit, of a trajectory to its frequency spectrum. As a result, jerk-limited trajectories are hand-tuned by trial-and-error to avoid unwanted vibrations, which is inefficient and limits achievable productivity.

In this paper, we present a novel technique to generate reference trajectories with optimal frequency spectra to avoid machine tool vibrations during linear point-to-point and spline interpolation. As opposed to tuning jerk parameters [2–4,9,10], or command pre-filtering [5–8], proposed technique is a direct approach. Reference acceleration profiles are generated so that their spectral energy is attenuated around vibration mode(s) of the machine. As a result, the necessity for jerk limitation or pre-filtering is eliminated, which unlocks a new potential to achieve better contouring accuracy, smoother surface finish and faster cycle times. Furthermore, to the authors' knowledge, demonstration of trajectory-induced vibrations on surface finish quality in actual machining experiments is a first.

2. Frequency optimal acceleration profiling

Reference trajectory induces machine tool vibrations in the form of inertial forced vibrations. The trajectory contains position, velocity, acceleration and jerk commands. Based on drive inertia, servo-motors deliver acceleration equivalent torque/force. This motor torque/force can directly excite any of lightly damped feed drive modes [1] and cause forced vibrations resulting path errors [6,9]. On the other hand, as the work-table accelerates its reaction, i.e. inertial, forces flow through the saddle and excite machine base and the column. As a result, low frequency “rocking” or “column” vibrations are induced that may deteriorate surface finish [5]. In any case, reference

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acceleration profile controls the excitation force. Therefore, its frequency spectrum must be attenuated around natural modes of the machine. This objective can be written by minimizing Discrete Fourier Transform (DFT) of acceleration, $A(kT_s)$:

$$\min J = \underbrace{\int_{\omega_1}^{\omega_2} \left|\sum_{k=0}^{N} A(kT_s)e^{-j\omega kT_s}\right|^2 d\omega}_{J_1} + \underbrace{\int_{\omega_3}^{\omega_4} \left|\sum_{k=0}^{N} A(kT_s)e^{-j\omega kT_s}\right|^2 d\omega}_{J_2} + \ldots \tag{1}$$

where J is the cost function and $\omega\in[\omega_1,\omega_2]\cup[\omega_3,\omega_4]\cup\ldots$ are target frequency bands. T_s is sampling time of servo loop, and $T_{acc} = NT_s$ is acceleration duration.

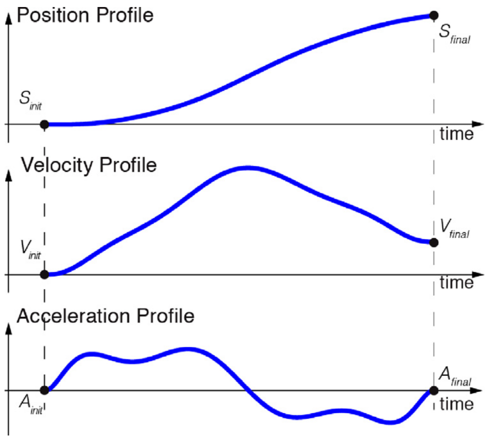

Fig. 1. Frequency optimal feed profile.

Modern NC systems have spline feedrate profiling functionality [10]. Reference trajectory is defined as a parametric spline with a *9th* order acceleration profile shown in Fig. 1:

$$\left.\begin{aligned} S(t) &= at^{11} + bt^{10} + ct^{9} \ldots + n \\ V(t) &= 11at^{10} + 10bt^{9} + \ldots + m \\ A(t) &= 110at^{9} + 90bt^{8} + \ldots + 2l \end{aligned}\right\} \tag{2}$$

where $\theta = [a, b, c, \ldots, n]$ contains the spline coefficients. The objective is to find the unknown trajectory coefficients θ that minimize the cost function J in Eq. (1). More importantly, this optimization problem must be solved in real-time. Let us first consider one of the sub-cost functions, e.g. J_1, and expand it:

$$\begin{aligned} J_1 &= \int_{\omega_1}^{\omega_2} \left(\sum_{k=0}^{N} A(kT_s)e^{-j\omega kT_s} \cdot \sum_{k=0}^{N} e^{j\omega kT_s} A(kT_s)\right) d\omega \\ &= \int_{\omega_1}^{\omega_2} \left(\boldsymbol{\theta} \boldsymbol{M}_{acc} \boldsymbol{E}\boldsymbol{E}^{*} \boldsymbol{M}_{acc}^{T} \boldsymbol{\theta}^{T}\right) d\omega \end{aligned} \tag{3}$$

where $\boldsymbol{E} = [1\ e^{-j\omega T_s}\ e^{-j2\omega T_s} \ldots e^{-(N-1)j\omega T_s}]^T$ is the Fourier Kernel, $\boldsymbol{E}^*$ is its complex conjugate, and $\boldsymbol{M}_{acc}$ is a constant obtained by writing the sampled acceleration profile in matrix form from Eq. (2) as:

$$\begin{aligned} A(kT_s) &= 110a(kT_s)^9 + 90b(kT_s)^8 + \ldots + 2l \\ &= \underbrace{[a\, b\, c \ldots n]}_{\theta} \underbrace{\begin{bmatrix} 0 & 110(T_s)^9 & 110(2T_s)^9 & \ldots & 110(NT_s)^9 \\ 0 & 90(T_s)^8 & 90(2T_s)^8 & \ldots & 90(NT_s)^8 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\times \boldsymbol{M}_{acc}} \end{aligned} \tag{4}$$

Next, taking frequency independent terms outside of the integral, Eq. (3) can be re-written in the following mathematically convenient quadratic form:

$$J_1 = \boldsymbol{\theta} \boldsymbol{M}_{acc} \underbrace{\left(\int_{\omega_1}^{\omega_2} (\boldsymbol{E}\boldsymbol{E}^{*}) d\omega\right)}_{\boldsymbol{F}} \boldsymbol{M}_{acc}^{T} \boldsymbol{\theta}^{\boldsymbol{T}} \tag{5}$$

Since we are only in interested in magnitude of DFT, $\overline{F} = \mathrm{Re}(F)$ is evaluated, and Eq. (5) is put in its final compact quadratic form as:

$$J_1 = \boldsymbol{\theta} K_{\boldsymbol{1}} \boldsymbol{\theta}^{\boldsymbol{T}} \tag{6}$$

where $K_{\boldsymbol{1}} = M\overline{F}M^T$ becomes a frequency dependent weight constant. By simply summing contribution of each frequency band, the global cost function (Eq. (1)) can be constructed:

$$K = K_{\boldsymbol{1}} + K_{\boldsymbol{2}} + \cdots + K_n \Rightarrow J = \boldsymbol{\theta} K \boldsymbol{\theta}^{\boldsymbol{T}} \tag{7}$$

As shown in Fig. 1, set of initial and final kinematic boundary conditions $S_{initi/final}$, $V_{init/final}$, $A_{init/final}$ needs to be introduced to control the motion. Boundary constraints are linear in spline parameters, and hence the problem of trajectory generation with optimal frequency spectra is postulated as:

$$\min \frac{1}{2} \theta K \boldsymbol{\theta}^T \quad \text{subject to}: \ \boldsymbol{L}\boldsymbol{\theta}^T - \boldsymbol{\zeta} = \boldsymbol{0} \tag{8}$$

where $\boldsymbol{L}$ and ζ are obtained from constraint evaluation. Note that the problem in Eq. (8) is similar to the minimum jerk trajectory generation problem in [2,3], except the fact that instead of minimizing jerk, proposed cost function directly penalizes frequency spectrum of acceleration. Compared to [11], Eq. (8) can accommodate multi-modes and can be extended to spline tool-paths as presented in Section 4. Above optimization problem is solved using the well-known Quadratic Programming (QP) technique by introducing Lagrange multipliers λ, and the spline trajectory parameters $\theta = [a,b,c,\ldots,n]$ are obtained analytically through solution of following linear equation system:

$$\left.\begin{aligned} K\boldsymbol{\theta}^T + \boldsymbol{\lambda}^T L &= 0 \\ L\boldsymbol{\theta}^T &= \boldsymbol{\zeta} \end{aligned}\right\} \Rightarrow \begin{bmatrix} K & L^T \\ L & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}^T \\ \boldsymbol{\lambda}^T \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\zeta} \end{bmatrix} \tag{9}$$

where $\boldsymbol{K}$ and $\boldsymbol{L}$ are 12×12 and 6×12 matrixes. Above solution can be sought either by direct matrix inversion, or making use of Gaussian elimination, both of which can be implemented conveniently in modern real-time numerical (NC) systems.

3. Generation of frequency optimal point-to-point (P2P) trajectories

Effectiveness of proposed trajectory generation technique is first validated during machining of P2P linear trajectories on a 3-axis micro-milling center shown in Fig. 2. *X-axis* is mounted on the base of the machine whereas column carries both *Y* and *Z* (spindle) axes. FRF between tool and workpiece is measured in *Z* direction through sinusoidal excitation delivered by *X* and *Y*-axes. As observed, both rocking mode at 20 Hz and column mode at 58 Hz may induce relative motion between tool and workpiece, and they can be excited by the rapid motion of *X* and *Y*-axes.

To avoid exciting both of those modes, proposed technique is designed to attenuate *2* freq. bands, $\omega_{n1} = [20,22]$ Hz and $\omega_{n2} =$

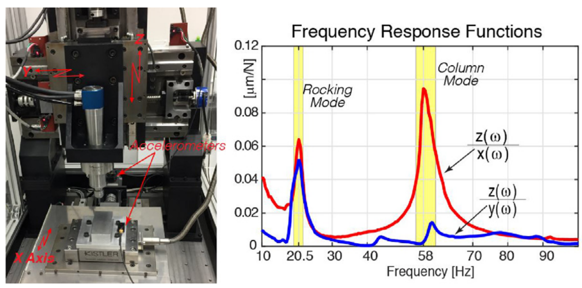

Fig. 2. 3-axis micro-milling center and its frequency response.

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