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Classical and quantum phenomenology in radiation by relativistic electrons in matter or in external fields

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ABSTRACT

Phenomenological aspects of radiation by relativistic electrons in external field, in matter or the vicinity of matter are reviewed, among which: infrared divergence, coherence length effects, shadowing, crystal-assisted radiation, quantum recoil and spin effects, electron side-slipping, photon impact parameter and tunneling in the radiation process.

1. Introduction

In Classical Electrodynamics (CED), a moving electron can emit radiation by two mechanisms:

(A) velocity change. The photon is directly emitted from the electron world line. This is the case of *Synchrotron Radiation* (SR), *Undulator Radiation* (UR), *Compton Back-Scattering* (CBS), *Bremsstrahlung* (BR), *Coherent Bremsstrahlung* (CBR) and *Channeling Radiation* (CR).

(B) transient medium polarization. If the motion is *ballistic* (i.e., rectilinear and uniform), but inside or near a medium, the photon is emitted by the polarization currents induced by the traveling Coulomb field of the electron. This is the case of *Cherenkov–Vavilov Radiation* (ČVR), *Transition Radiation* (TR) [in optical (OTR) or X-ray (XTR) domains], *Diffraction Radiation* (DR), *Smith–Purcell Radiation* (SPR), *Parametric X-Rays* (PXR) and *Polarization Bremsstrahlung* on individual atoms. Mechanisms (A) and (B) can coexist, e.g., in Diffracted Channeling Radiation.

A vast theoretical and experimental work is currently done on these radiations, specially since the prediction of intense Channeling Radiation by relativistic electrons by Muradin Kumakhov [1]. In this paper we review some of their "universal" properties in a phenomenological approach. We gather these properties under three headings:

- **Classical radiation** (Section 2), for photon energy ω much smaller than the electron energy ϵ;
- **Hard photon emission** (Section 3), for $\omega \sim \epsilon$;
- **Impact parameter properties** (Section 4).

We will assume that the electron is ultrarelativistic ($\epsilon/m_e \equiv \gamma \gg 1$) and behaves classically, at least between the photon emissions. We will not treat its dynamics in the "radiator" (UR device, amorphous matter, crystal, TR or PXR targets, etc.) which, in the crystal case, involves many phenomena: channeling, dechanneling, volume capture, volume reflection, etc. We suppose that the trajectory of the electron has been calculated beforehand. For the general theory of radiation the reader may consult [2–4] and for BR, CR and CBR, [5–8]. Many considerations presented below can be found in [9,10].

We work with natural unit systems where $\hbar = c = 1; \alpha = e^2/(4\pi) \simeq 1/137$; the letter "$\gamma$" can also designate a photon, of momentum $\mathbf{k} = \omega\mathbf{n}; n = |\mathbf{n}|$ is the refraction index. The components $X_\parallel$ and $\mathbf{X}_\perp$ of a vector $\mathbf{X}$ are parallel and perpendicular to $\mathbf{k}$, whereas X_L and $\mathbf{X}_T$ are relative to the electron velocity $\mathbf{v}(t)$ or its mean value $\langle\mathbf{v}\rangle$.

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2. Classical radiation

We consider an electron of classical trajectory $\mathbf{r} = \mathbf{r}(t)$ and suppose that the emission of one photon does not modify the trajectory noticeably and does not involve the electron spin. This excludes channeling at low energy ($\epsilon \lesssim 100$ MeV), where the number of transverse energy states is low, and at very high energy, where hard photon emission ($\omega \sim \epsilon$) takes place. For a given polarization $\hat{e}$, the photon spectrum writes

$$dN(\hat{e}) = \frac{\alpha}{4\pi^2} \frac{d^3\mathbf{k}}{\omega} |\mathcal{A}|^2. \tag{1}$$

For mechanism (A) in vacuum, $\mathcal{A}$ is given in covariant and gauge-invariant way by (see, e.g., Eq. (1.23) of [7])

$$\mathcal{A} = -\hat{e}^* \cdot A(k), \quad A = \int_{\mathcal{T}} dX \exp(i\phi), \quad \phi = k \cdot X. \tag{2}$$

$X = (t, \mathbf{r})$, $k = (\omega, \mathbf{k})$, A and $\hat{e}$ are 4-vectors. We take the metric where $k \cdot X = \omega t - \mathbf{k} \cdot \mathbf{r}$, $\hat{e}^* \cdot \hat{e} = -1$. The integral in (2) is along the whole electron trajectory $\mathcal{T}$. In a non-covariant formulation we take the gauge $\hat{e}^0 = \hat{\mathbf{e}} \cdot \mathbf{k} = 0, \hat{\mathbf{e}}^* \cdot \hat{\mathbf{e}} = 1$, and replace $-\hat{e}^* \cdot A$ by $\hat{\mathbf{e}}^* \cdot \mathbf{A}_\perp$ with

$$\mathbf{A} = \int_{-\infty}^{\infty} dt_\mathrm{d} \exp(i\phi) \frac{d\mathbf{r}}{dt_\mathrm{d}} = \frac{i}{\omega} \int_{-\infty}^{\infty} dt_\mathrm{d} \exp(i\phi) \frac{d^2\mathbf{r}}{dt_\mathrm{d}^2}, \quad \phi = \omega t_\mathrm{d}. \tag{3}$$

$t_\mathrm{d} = t - \mathbf{n} \cdot \mathbf{r}$ is the *detection time* up to a constant. The integral can be defined by inserting an extra factor $\exp(-\varepsilon|t_\mathrm{d}|)$ and letting $\varepsilon \to +0$. The result converges if $\mathbf{v}(t)$ does not oscillate indefinitely. $d\mathbf{r}_\perp/dt_\mathrm{d}$ and $d^2\mathbf{r}_\perp/dt_\mathrm{d}^2$ are the *apparent* perpendicular velocity and acceleration. Their is no bound for $d\mathbf{r}_\perp/dt_\mathrm{d} = \mathbf{v}_\perp/(1 - \mathbf{n} \cdot \mathbf{v})$, unlike the $|\mathbf{v}| < 1$ one. The second expression of (3) emphasizes the role of the acceleration.

The photons emitted by one classical trajectory are fully polarized and form a *coherent state* [11]. Their number in any $\mathbf{k}$ domain follows a Poisson distribution.

For mechanism (B), or a coexistence of (A) and (B), $\mathcal{A}$ can be calculated using the reciprocity theorem, which is related to time reversal. It gives [9]

$$\mathcal{A} = -\int_{\mathrm{rev}(\mathcal{T})} d\mathbf{r}' \cdot \mathbf{E}^{(\mathrm{in})}_{-\mathbf{k},\hat{\mathbf{e}}^*}(t, \mathbf{r}'), \tag{4}$$

where rev($\mathcal{T}$) is the time-reversed trajectory $\mathbf{r}'(t) = \mathbf{r}(-t)$ of the electron; $\mathbf{E}^{(\mathrm{in})}_{-\mathbf{k},\hat{\mathbf{e}}^*}$ is the complex electric field of the *ingoing* solution of the homogenous Maxwell equations in matter, for a wave coming from the detector with momentum $-\mathbf{k}$ and polarization $\hat{\mathbf{e}}^*$. It is normalized to $\hat{\mathbf{e}}^* \exp(-i\omega t - i\mathbf{k} \cdot \mathbf{r})$ in vacuum in the detector region. Eq. (4) is well suited to TR, SPR and PXR. It takes the absorption in the radiator into account if $\mathbf{E}^{(\mathrm{in})}$ is calculated with a complex refraction index n.

In the following we will use (3) in the ultrarelativistic and small-angle approximations $\gamma \gg 1, |\mathbf{v}| \simeq 1, |\mathbf{v}_\perp(t)| \ll 1, |\mathbf{n}_\mathrm{T}| \ll 1$. Then

$$dt_\mathrm{d}/dt = 1 - \mathbf{n} \cdot \mathbf{v} \simeq [\gamma^{-2} + \theta^2 - \delta n^2(\omega, \mathbf{r})]/2 \ll 1, \tag{5}$$

with $\vec{\theta} \equiv \mathbf{n}_\mathrm{T} \simeq -\mathbf{v}_\perp(t) \simeq \mathbf{n} - \mathbf{v}(t)$ and $\delta n^2 \equiv n^2 - 1$. The δn^2 term allows to extend (3) to an electron moving in a transparent stratified medium of index n close to 1, if one neglects the variations of $\mathbf{n}_\mathrm{T}$ due to the refractions. This is the case for XTR ($\delta n^2 \simeq -\omega_\mathrm{P}^2/\omega^2$) at non-grazing incidence ($|\mathbf{k} \cdot \hat{\mathbf{m}}| \gg \omega_\mathrm{P}$, where ω_P is the plasma frequency and $\hat{\mathbf{m}}$ the unit vector normal to the medium boundary). When in doubt, use (4).

Sudden velocity change. As a prototype of mechanism (A), we consider an electron of trajectory $\mathbf{r} = \mathbf{v}_\mathrm{i}\, t$ for $t < 0$ and $\mathbf{r} = \mathbf{v}_\mathrm{f}\, t$ for $t > 0$ in vacuum. Then,

$$\mathbf{A}_\perp = \mathbf{A}_\perp(\mathbf{v}_\mathrm{i}, \mathbf{k}) - \mathbf{A}_\perp(\mathbf{v}_\mathrm{f}, \mathbf{k}), \tag{6}$$

with

$$\mathbf{A}_\perp(\mathbf{v}, \mathbf{k}) = (i\omega)^{-1} \mathbf{v}_\perp/(1 - \mathbf{n} \cdot \mathbf{v}) \simeq (2i/\omega)\, \vec{\theta}/(\gamma^{-2} + \theta^2). \tag{7}$$

In soft Bremsstrahlung on one atom, $|\mathbf{v}_\mathrm{i}| \simeq |\mathbf{v}_\mathrm{f}|$. For $\mathbf{v}_\mathrm{f} = 0$ we have a *suddenly stopped* electron, for $\mathbf{v}_\mathrm{i} = 0$ a *suddenly starting* electron (e.g., in beta decay or Compton scattering). They emit the same photon spectrum,

$$dN(\vec{\varepsilon}) = \frac{\alpha}{\pi^2} \frac{d\omega}{\omega} d\Omega \left| \frac{\vec{\theta} \cdot \vec{\varepsilon}}{\gamma^{-2} + \theta^2} \right|^2, \tag{8}$$

but with opposite amplitudes. It can be understood with the superposition principle: to a suddenly stopping e^-, we can add - in thought – an e^+ co-moving with the e^- at $t < 0$, but not stopping. This does not add or subtract any radiation, but the new system is equivalent to a suddenly starting e^+.

The spectrum (8) has the following "universal" properties:

- an annular-lobe angular distribution peaked at $\theta = 1/\gamma$,
- a radial polarization (*i.e.*, in the $(\mathbf{v}, \mathbf{n})$ plane),
- an infrared divergence $dN/(d\omega d\Omega) \sim \omega^{-1}$, related to the semi-infinite ballistic motion,
- an ultraviolet divergence, related to the infinite acceleration at $t = 0$. It does not actually happen, because $\omega < \epsilon$.

Whether the velocity change is sudden or not, Eqs. (6, 7) give the infrared part of the spectrum when the motion is ballistic for $t \to \pm\infty$. In large-angle scattering ($|\mathbf{v}_\mathrm{i} - \mathbf{v}_\mathrm{f}| \gg \gamma^{-1}$) we have two well-separated annular lobes. For the *dipolar* regime $|\mathbf{v}_\mathrm{i} - \mathbf{v}_\mathrm{f}| \ll \gamma^{-1}$, the interference between the lobes is essential. The transition between the two cases is discussed in [12].

The annular lobe (8) appears in many other experimental situations: in backward OTR from a planar surface, in forward OTR from any opaque (polished or rough) screen, in the *edge radiation* from a bending magnet, etc., although the finite size $L_{\max}$ of the straight beam line provides an infrared cutoff, $\omega_{\min} \sim 2\,[L_{\max}(\gamma^{-2} + \theta^2)]^{-1}$ (see the discussion after Eq. 11).

In simulations, one usually truncates the integral in (3). Doing so in the second expression, we introduce a spurious infrared divergence. It is like completing the trajectories by semi-infinite ballistic motions. A phenomenological infrared cutoff has to be applied; in [13] we imposed a minimum increase of ϕ in the truncated piece: $\Delta\phi > 0.5$ radian; the coefficient 0.5 was such as to satisfy a filtered sum rule [14]. Truncating in the first expression of (3) produces a spurious ultraviolet divergence, as if the electron started and stopped suddenly.

Longitudinal coherence (LCO) [15]. The *coherence lenght* L_coh is most often defined as the path of trajectory over which ϕ increases by 1 radian. In the X-ray domain and for a straight trajectory in uniform medium,

$$L_\mathrm{coh}(\mathbf{v}, \mathbf{k}) = dt/d\phi \simeq (2/\omega)\,(\gamma^{-2} + \theta^2 + \omega_\mathrm{P}^2/\omega^2)^{-1}. \tag{9}$$

It can reach macroscopic values. For a *curved* trajectory, the lenght of a $\Delta\phi = 1$ arc depends on its middle point. We choose the latter to be such that θ is minimum. It gives

$$L_\mathrm{coh} \simeq \min\{(2/\omega)\,(\gamma^{-2} + \psi^2)^{-1}, (24R^2/\omega)^{1/3}\}, \tag{10}$$

ψ being angle between the photon and the curvature plane.

If the electron undergoes two successive scattering $\mathbf{v}_\mathrm{i} \to \mathbf{v}$ at $X = (t, \mathbf{r})$ and $\mathbf{v} \to \mathbf{v}_\mathrm{f}$ at $X' = (t', \mathbf{r}')$, the $[XX']$ path contributes to the first integral (3) by