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A hybrid method based on reduced constraint region and convex-hull edge for flatness error evaluation

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ABSTRACT

A new hybrid approach is proposed for evaluation of flatness error using the Minimum Zone Method. The reduced constraint region is used to rapidly determine the effective direction of enveloping planes, and the convex-hull edge of that direction is used to obtain the minimum zone solution through iteration. The proposed method is validated through the numerical tests with a number of test data sets including those published in literatures and large new data sets of actual measurements. The computed results indicate that an exact and fast minimum solution can always be obtained using the proposed method. It is therefore concluded that the proposed method is one of the approaches which can be used to further improve the accuracy and efficiency of flatness error evaluation.

1. Introduction

Plane feature is one of the most basic forms of geometric primitives. It is specified in ISO [1,2], ANSI [3] standards and CIRP Encyclopedia of Production Engineering [4] that there are two flatness error evaluation methods of least squares method (LSM) and minimum zone method (MZM). It is recommended in ISO standards to get form tolerance evaluated on the basis of MZM. MZM is defined as two parallel planes enclosing the flatness surface and having the least separation. However it has not been spelt out explicitly in the standards how to determine minimum zone. Conversely, LSM is univocally defined by its mathematical formulation, which implies minimizing the L_2 norm of the error vector of measured points. The requirement of ISO standards is not satisfied, because the L_∞ norm is required to be minimized. So the results of LSM are inconsistent with the MZM solution. It is impossible to predict the difference between them in practice. And MZM responds better on flatness tolerance evaluation.

Generically, final inspection results obviously depend on actual measurements, sampling strategy and method of flatness tolerance evaluation. The accuracy and rational distribution of sample points are the precondition of error evaluation, which need to be complete, continuous and homogeneous as far as possible. And the number of measured data points is a predominant factor associated with uncertainty [5,6]. Therefore, large-sized data sets are preferred to ensure the desired evaluation accuracy. Due to the relationship of the number of points with measuring cost and time, some works [7,8] have sought the reduction of sampling size for a desired precision. Another way is to use more efficient measurement devices. Non-contacting measurement equipment has replaced contacting probes used for inspection of thin or easily deflected work pieces. The advantage is a large number of measured data points can be generated within a reasonable period of time. Therefore the evaluation algorithms are required to be more effective and reliable in handling these large-sized samples.

The methods which have been developed for flatness tolerance evaluation so far can be mainly classified into numerical approaches and computational geometry approaches. Numerical approaches can be further divided into such specific categories as nonlinear optimization based search methods, linear approximation methods, and meta-heuristic methods.

Much work has been done in recent years on the nonlinear optimization model based search methods. For example, Kaiser and Suzuki [9] applied downhill simplex search and repetitive bracketing method. Similar to Kaiser and Suzuki, a method is used where the least-squares plane is used as a starting point, and in a downhill simplex method the gradient in x- and y direction is varied until a minimum flatness value is found [10]. Prakasvudhisam [11] showed that support vector regression (SVR) method could be used to solve the flatness problem. One drawback of nonlinear

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search methods is the non-convexity of optimization problem and the need of several trials with different initial conditions to ensure global optimum. Approximation methods have been tried to improve computational efficiency. Cheraghi [12] proposed a linear search to solve nonlinear problem. Weber et al. [13] considered the plane equation represented by two angle parameters and one intercept constant, whose linearization by Taylor expansion. Meta-heuristics have been applied to minimum zone to solve combinatory optimization problems in an efficient way. In this category we find the improved genetic algorithm by Wen [14] and Tseng [15], particle swam optimization by Kovvur [16], and differential evolution algorithm by Wang [17]. They used different intelligence algorithms to solve a global optimization problem.

Much work has also been done on computational geometry approaches. Geometrical characteristics of a data set can be expressed by a convex hull which is defined as the smallest convex domain including all data points. Computational geometry methods determine the minimum width of convex hull to solve the flatness problem. For example, Huang [18] reduced the calculation of large number of data points and constructed a new convex hull by adding new points. Samual and Shunmugam [19] proposed algorithms to evaluate all the antipodal pairs of face and vertex alone. Hermann [20] proposed an incremental convex hull algorithm for the evaluation of a convex hull. Lee [21,22] presented a convex-hull edge method which considered 2–2 and 3–1 models to guarantee an exact minimum zone (MZ) solution.

Some authors have tried to adopt other approaches for flatness tolerance evaluation. For example, Liu [23] combined genetic algorithm (GA) with geometric calculation. Calvo et al. [24] suggested the hard minimax problem into minisum problem and finally to eigenvector problems. Zhu and Ding [25] proposed the equivalence between the width of a point set and the inner radius of convex-hull. Deng [26] presented valid characteristic point method (VCPM) by which a normal vector of ideal enveloping planes satisfying the MZ criteria is iteratively search for. However, with the fast development of precision measuring instrument industry in recent years, the requirements for accurate and reliable measurements become increasingly stringent in reality. And the speed of calculation also becomes increasingly important, not so much for traditional flatness applications in precision such as low volume optics or artifact surfaces, but for the growing trend toward optical inspection of manufactured work pieces giving dense data sets. Meantime the high volume production cases where the measurement and analysis must keep pace with the speed of the production. It is of great significance to find ways and means by which due considerations can be given to both measuring accuracy and speed at the same time in practice.

Therefore, a new hybrid approach is proposed for evaluation of flatness error using the Minimum Zone Method. The reduced constraint region is used to rapidly determine the effective direction of enveloping planes, and the convex-hull of that direction is used to obtain the minimum zone solution through iteration.

2. Mathematical model for flatness error evaluation

2.1. Mathematical model for MZ flatness evaluation

The purpose of minimum zone method is to search for the normal vector of enveloping planes. If $P_i(x_i, y_i, z_i)$ $(i = 1, 2, \ldots, n)$ is the measured point extracted by an actual plane. The flatness tolerance is two parallel planes enveloping all the data points of a surface, and the minimum separation between two parallel planes is the minimum zone solution of flatness error.

If the reference plane equation of two parallel planes is

$$m = ax + by + cz \tag{1}$$

The condition of satisfaction can be expressed as

$$a^2 + b^2 + c^2 = 1 \tag{2}$$

Distance d_i from data point $P_i(x_i, y_i, z_i)$ to parallel plane can be expressed as

$$d_i = m - ax_i - by_i - cz_i \tag{3}$$

The minimum separation between two parallel planes is equivalent to minimax distance f between points and reference plane which can be expressed as shown below.

$$\begin{aligned} f &= \min\left\{ \max_{\substack{i=1:n\\ j=1:n}} \left|d_i - d_j\right| \right\} \\ &= \min\left\{ \max_{\substack{i=1:n\\ j=1:n}} \left|(m - ax_i - by_i - cz_i) - (m - ax_j - by_j - cz_j)\right| \right\} \\ &= \min\left\{ \max_{\substack{i=1:n\\ j=1:n}} \left|(ax_i + by_i + cz_i) - (ax_j + by_j + cz_j)\right| \right\} \\ &= \min\left\{ \max_{\substack{i=1:n\\ j=1:n}} \left|a(x_i - x_j) + b(y_i - y_j) + c(z_i - z_j)\right| \right\} \end{aligned} \tag{4}$$

Obviously, minimum distance f is the function of (a, b, c). Consequently, the evaluation of flatness error is to search for the values of (a, b, c), which is a nonlinear optimization problem.

2.2. Geometric characteristic points model for flatness error evaluation

The MZ solution of flatness error has been verified and adopted in studies [27–29]. The parallel planes enveloping all the data points must satisfy the following conditions: (1) As shown in Fig. 1, at least four points must be in contact with the two enclosing parallel planes in the form of a 3-1 model or a 2-2 model. In the 3-1 model, there are three points on the upper plane, one point on the lower plane, and the projection of the low point is within the triangle composed by the three high points, or vice versa. In the 2-2 model, there are two points on the upper plane, two points on the lower plane, and the line of two high points crosses the line of two low points when they are projected onto the upper or lower plane.

(2) Another phenomenon should be noted here [27]. In the 3-1 model, the normal vector of the parallel planes is determined by three points, and one point outside is needed to determine the parallel planes. In the 2-2 model, a plane is the vectorial subspace of dimension two, so the parallel planes can be defined by two nonlinear vectors. As shown in Fig. 2, two parallel planes become two straights outside after being projected along the direction of arrow, and the distribution of data points will be directly coincident with the MZ criteria of straightness.