Mechanism and Machine Theory 94 (2015) 41–63

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Dynamic and energetic characteristics of a tri-stable magnetopiezoelastic energy harvester

Pilkee Kim, Jongwon Seok [*]

School of Mechanical Engineering, College of Engineering, Chung-Ang University, 84 HeukSeok-Ro, DongJak-Gu, Seoul, 156-756, Republic of Korea

ARTICLE INFO

Article history:
Received 21 November 2013
Received in revised form 10 July 2015
Accepted 4 August 2015
Available online 24 August 2015

Keywords:
Tri-stable energy harvester
Magnetic repulsion
Pitchfork bifurcation
Degenerate bifurcation
Bifurcation set diagram
Interwell motion

ABSTRACT

In this study, a mathematical model of a tri-stable energy harvester is developed and used to investigate its formation mechanisms for multi-stability states, nonlinear dynamic behaviors, and power generation performance. Bifurcation analyses for the equilibrium solution of the derived model system are performed. It is shown that the present energy harvester system can exhibit multi-stable (mono-, bi-, and tri-stable) behaviors depending on the two geometric parameters associated with the locations of the tip and external magnets. It is also found that the tri-stability is initiated by a new pitchfork bifurcation or a degenerate pitchfork bifurcation that leads to a pair of saddle-node bifurcations. Bifurcation set diagram is obtained in the parametric space of these two geometric parameters, which can be used to design the potential wells of the multi-stable energy harvesters. Potential energy diagrams are also obtained and they show that the distance between the outer potential energy wells for the tri-stable state is formed in a way that it is larger than that for the equivalent bi-stable state. A series of numerical simulations performed on the present system well illustrates the high output power generation characteristics of the tri-stable energy harvester over a broad operating frequency band.

1. Introduction

An energy harvester is, in general, an electromechanical device with which a variety of ambient energies, usually unused and disposed, are harvested, and then converted and stored in a capacitor or a rechargeable battery in the form of electrochemical energy. Recently, such an energy harvester has been developed and become a key element for satisfying the strong demand for a means to supply power to small electronic devices without external electric sources or internal batteries. This is because the use of batteries naturally entails several shortcomings such as short lifetime, contribution to environmental damage, and a requirement of frequent maintenance. In realizing such self-powered electronic systems, they can be made through mass production and installed on a large scale. Furthermore, these systems are better for use in dangerous locations because they are virtually maintenance-free after their deployment (that is, they are "deploy and forget" [1]). Furthermore, the energy harvesters can be a good alternative for eliminating or reducing the needs for batteries in self-powered systems. Nowadays, the energy harvesting techniques are widely used for various applications including ubiquitous sensor networks, wireless sensor nodes or transmitters, and intelligent RFID platforms [1]. The rapid increase of current interest in such application fields has driven an intense demand for the development of high-efficiency energy harvesting techniques that can extract energy from various external sources effectively.

For the last two decades, a number of studies have been performed on methods for effectively harvesting energy from a variety of external sources (vibration, light, heat, sound, etc.). Among these energy sources, vibration energy is at the focal point of the attention of many researchers because it exists almost everywhere, and is easily convertible to electric energy. Therefore, an energy harvester

---

[*] Corresponding author. Tel.: +82 2 820 5729; fax: +82 2 3280 9982.
E-mail address: seokj@cau.ac.kr (J. Seok).

http://dx.doi.org/10.1016/j.mechmachtheory.2015.08.002

that extracts vibration energy can provide a convenient, useful, and efficient means to supply power to a small electronic device open to the vibratory environment [1–3]. Most of the first generation of vibration-energy harvesters were based on some form of linear resonator and used their resonant characteristics of a strong resonant peak, albeit with a very narrow bandwidth. These linear energy harvesting resonators can exhibit a satisfactory performance with a large vibration amplitude only when the excitation frequency and the system's natural frequency are synchronized (i.e., when a resonance occurs). One of the most substantial and inevitable problems of such a linear energy harvester is the fact that its energy harvesting efficiency becomes significantly lowered in the off-resonant frequency region. To overcome such a deficit in efficiency, various active tunable energy harvesters, equipped with several frequency tuning techniques, have been proposed [4–6]. However, such an active system naturally possesses kinematical and dynamical constraints due to the complexity in its structure. More importantly, it fails to sufficiently improve the energy harvesting efficiency [3].

Another alternative for improving the efficiency is to use the resonant characteristics of a nonlinear oscillator such as the stiffness softening or hardening effect, with which the effective resonant frequency band can be widened [7–9]. In general, the primary resonant peak of a linear system is formed with a narrow bandwidth, but that of a nonlinear system can be made with a relatively wider bandwidth, albeit accompanying a hysteretic phenomenon that has a dependency on the sweep direction of the excitation frequency. Accordingly, building a nonlinear vibration-energy harvester with the increased available resonant frequency bandwidth may be a good way to improve the energy harvesting efficiency of a linear system.

Recently, a bi-stable energy harvester has been proposed as an efficient vibration harvester that can extract the vibration energy over a wide frequency band [10]. A beam-type bi-stable energy harvester (known as the post-buckled beam) can be realized by imposing an axial force on the beam tip using a magnetic [11] or mechanical [12] force. Such a post-buckled cantilever energy harvester can be simplified into a lumped parameter model with negative stiffness, i.e., a forced Duffing equation [9]. Actually, a forced Duffing equation with a negative stiffness is often inaccurate in the exact prediction of the dynamics of bi-stable energy harvesters but it is useful to theoretically explain their fundamental dynamic characteristics. With this equation, it is found that the bi-stable system possesses a potential energy function with symmetric double wells; therefore, the system is basically unstable at its central equilibrium point. When the excitation force exerted on the base of the system exceeds a critical value, the system's kinetic energy becomes large enough to cross over the two potential energy wells, resulting in the generation of interwell motion with a large amplitude. Although the initiation of such interwell motion is known to help the system to harvest vibration energy over a wide frequency range [11], the interwell motion is also known to be highly sensitive to the excitation force strength [12].

To investigate the dynamic characteristics and related energy harvesting performance of the bi-stable energy harvesters, many theoretical and experimental studies have been conducted [11–22]. Cottone et al. [11] illustrated that the power generated by a bi-stable energy harvester with a band-limited noise excitation can be improved up to 400%–600% in comparison with a linear resonator under a similar situation. Ferrari et al. [14] evaluated the broadband energy harvesting characteristics of a bi-stable energy harvester that used the magnetic repulsion force under white-noise excitation. Also, they proposed the possibility of miniaturizing the bi-stable energy harvester by fabricating a MEMS U-shaped cantilever-beam harvester. Stanton et al. [15] studied, both theoretically and experimentally, the nonlinear response characteristics of a bi-stable oscillator under swept sine excitations, and evaluated the resulting energy harvesting performances. On the other hand, Mann et al. [12] proposed a bi-stable electromagnetic-induction energy generator that used a non-contact magnetic repulsion force, and they evaluated its displacement response and generated power under chirp excitations.

The above-mentioned energy harvesters basically use the magnetic repulsion effect between two permanent magnets. In contrast, studies on the bi-stable energy harvesters that use the magnetic attraction effect are also underway. Ferrari et al. [16] proposed a single-magnet bi-stable harvester composed of a ferromagnetic cantilever beam, and showed that the effective voltage generated by its interwell motion can reach up to four times higher than the equivalent linear harvester. Erturk and Inman [17] investigated, both theoretically and experimentally, the high-energy orbit of a bimorph energy harvester composed of a bi-stable cantilever structure, known as the Moon beam [18]. Later, Erturk et al. [19] demonstrated that the working frequency band of the bi-stable energy harvester can be extended by adjusting the position angle of the source magnet. In addition, the bi-stable energy harvesters that use the mechanically buckled beam or plate are also being studied [12,20–22]. It is known that such a mechanical bi-stable energy harvester has the advantages of simple structure and high output power density.

In this study, a series of mathematical modelings and numerical simulations is performed in order to investigate the dynamic characteristics of a tri-stable bimorph cantilever energy harvester and its energy harvesting performance. The tri-stability of this system can be realized through the repulsion force acting on the permanent magnet located at the tip of the cantilever beam due to the two external magnets facing the tip magnet with the same polarity. Unlike the bi-stable energy harvester mentioned above, the interwell width of the present tri-stable system can be controlled to a certain degree by changing the gap distance between the two external magnets. Although such a tri-stability was used for signal amplification/filtering purposes [23], a tri-stable energy harvester has not been reported thus far.

The present study proposes a new type of a tri-stable energy harvester in search of an energy harvester with an enhanced energy harvesting performance compared to existing bi-stable energy harvesters. The conventional bi-stable energy harvesters were well-known to generate electric power in a broad frequency band using large-amplitude interwell motions at the expense of ambient excitation strength that is greater than the threshold value necessary to induce the potential well escape phenomenon [2]. The proposed tri-stable system can have shallower and wider potential wells than the bi-stable energy harvester that enable it to harvest electric energy in a broader frequency band, even under a weaker ambient excitation. Therefore, the present tri-stable system can be a promising alternative that mitigates the limitations of the existing bi-stable energy harvester and improves the energy harvesting performance. The main objective of the present study is to theoretically investigate and provide more physical insight into the underlying of nonlinear characteristics and potential merits of such a tri-stable energy harvester.