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Modelling transverse matrix cracking and splitting of cross-ply composite laminates under four point bending

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ABSTRACT

The transverse matrix cracking and splitting in a cross-ply composite laminate has been modelled using the finite element (FE) method with the commercial code Abaqus/Explicit 6.10. The equivalent constraint model (ECM) developed by Soutis et al. has been used for the theoretical prediction of matrix cracking and results have been compared to those obtained experimentally and numerically. A stress-based traction–separation law has been used to simulate the initiation of matrix cracks and their growth under mixed-mode loading. Cohesive elements have been inserted between the interfaces of every neighbouring element along the fibre orientation for all 0° and 90° plies to predict the matrix cracking and splitting at predetermined crack spacing based on experimental observations. Good agreement is obtained between experimental and numerical crack density profiles for different 90° plies. In addition, different mechanisms of matrix cracking and growth processes were captured and splitting was also simulated in the bottom 0° ply by the numerical model.

1. Introduction

Advanced fibre reinforced composites have been widely used in industry, especially to replace conventional metal alloys in aerospace applications for weight saving and improvement of mechanical performance [1]. However, structural components made from composites exhibit a relatively brittle behaviour and poor damage resistance when they are subjected to tensile, compressive or mixed-mode loading conditions, which may present critical issues for their implementation, with potentially catastrophic consequences [1–3]. Damage of composite laminates is a complicated process which is accompanied by internal fibre and matrix failure that appears within individual plies and results in interlaminar cracking (delamination). In general, the transverse matrix cracking in off-axis plies of a laminate has been recognized as the first damage mode occurring in composite laminates due to its resin-dominated characteristics [4]. This kind of damage usually does not give rise to ultimate failure but can reduce the stiffness and strength of composites and lead to other damage modes such as delamination. This develops due to the stress concentration at the crack tip at the ply interface, which may result in complete loss of load-carrying capability of the laminate [5–9]. Thus it is crucial to understand internal damage mechanisms in composite laminates at an earlier stage of design.

Finite element (FE) techniques are increasingly employed to accurately simulate composite damage within a defined constitutive model, which effectively reduces the need for costly physical tests. A large body of research has been undertaken to develop numerical modelling of damage in composites using the FE method. Hashin proposed a method to separately model various fibre and matrix damage modes [10,11] within individual plies, which was further developed by Naim [12], Varna [13] and Berglund [14,15]. For cross-ply laminates, Soutis et al. further developed the equivalent constraint model (ECM) to predict the stiffness reduction and matrix cracking due to crack induced delamination based on shear-lag analysis [4–9,16,17]. Continuum damage mechanics (CDM) has also been a popular means of modelling damage in composite laminates [18–21]. Shi and Soutis [22] combined different damage modes characterised by initiation and evolution criteria with nonlinear shear behaviour in a composite laminate and implemented them into a finite element code to simulate impact induced damage. This yielded a good prediction for matrix cracking and delamination was accurately captured by the damage model. The damage criteria used in previous studies were able to approximately capture the topology and extent of a damaged area. However, individual transverse matrix cracks or splits were rarely modelled successfully within the single ply or at the fibre/matrix interface.

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A numerical method was developed for this current study using cohesive elements with traction–separation damage criteria to simulate the matrix cracking and splitting in each individual lamina. The cohesive elements were placed between neighbouring elements parallel to the fibre direction in 0° and 90° plies. Equally spaced cracks have been assumed in the model based on experimental observation, as shown in Fig. 1. A general contact algorithm was used with pre-defined contact properties for the contact of rigid punches, supporters, laminate plates and interior surfaces inside the laminate. The numerically obtained crack density of different 90° plies are then compared to experimentally measured and ECM theoretically predicted values.

2. Theoretical model

2.1. Equivalent constraint model

The equivalent constraint model (ECM) is a theoretical approach used to predict transverse matrix cracking in multidirectional laminates under multiaxial in plane loading and a brief description of main assumptions and simplifications are discussed here for the reader's benefit. It is assumed that cracks in a damaged lamina are uniformly spaced, which is crucial to solving problems by analysis of a representative volume element. A schematic of the typical ECM representation with a damaged lamina is shown in Fig. 2. The layer, k denotes the damaged lamina and all plies above and below the kth ply are replaced with homogeneous layers (I and II), which are governed by the equivalent constraint effect. The stiffness properties of equivalent constraint layers can be obtained by the laminate plate theory (LPT), which provides the stress and strain relationship.

Due to the symmetry of a $[\pm\theta_m/90_n]_s$ laminate, as shown in Fig. 1 (for a $[0/90]_s$ lay-up), the analysis is reduced to one quarter of the representative segment. Matrix cracking in the 90° ply is expected to be the first damage mode to occur. Stresses can be calculated from the stiffness of the constrained homogeneous layers and the modified stiffness of the cracked ply. In order to determine stresses in the damaged ply, it is assumed that the total strain in the individual lamina is equivalent to that in the laminate (implying continuity). This is given by,

$$\bar{\varepsilon}_i^{(k)} = \bar{\varepsilon}_i \quad k = 1, 2 \tag{1}$$

$\bar{\varepsilon}_i^{(k)}$ and $\bar{\varepsilon}_i$ denote the total strain vectors of the kth layer and laminate, respectively. Thus, the average constitutive equations of a damaged lamina can be expressed as:

$$\bar{\sigma}_i^{(k)} = Q_{ij}^k(\bar{\varepsilon}_j + \bar{\varepsilon}_j^{0(k)}) \quad k = 1, 2 \tag{2}$$

where $\bar{\sigma}_i^{(k)}$ represent the total stress vector of the constraint layers ($k = 1$) and the damaged 90° ply ($k = 2$), respectively. $\bar{\varepsilon}_j^{0(k)}$ is the residual thermal strain vector of the kth layer. Q_{ij}^k is the stiffness of the constraining layers ($k = 1$) and modified reduced stiffness of the damaged 90° ply ($k = 2$). The reduced stiffness matrix of the damaged ply can be derived by the in-situ damage effective function (IDEF), which the detailed equations can be studied in Ref. [16] and won't be introduced here.

Then the laminate stress can be written using the classical laminate plate theory:

$$\bar{\sigma}_i = \frac{1}{(1+\chi)}(\bar{\sigma}_i^{(2)} + \chi\bar{\sigma}_i^{(1)}) \tag{3}$$

where χ is the thickness ratio of the constraining layer over the thickness of the 90° layer. The constitutive relation of the cracked laminate is obtained by combining Eqs. (2) and (3)

$$\bar{\sigma}_i = \bar{Q}_{ij}(\bar{\varepsilon}_j - \bar{\varepsilon}_j^p) \tag{4}$$

where $\bar{Q}_{ij}$ is the in-plane stiffness matrix of the damaged laminate. $\bar{\varepsilon}_j^p$ is a permanent strain, which represents the effect of interaction of damage and residual stresses and is defined as:

$$\bar{\varepsilon}_j^p = -\bar{S}_{ij}\frac{1}{(h_1+h_2)}\sum_{k=1}^{2} h_k Q_{jl}^k S_{lm}^{0(k)} \bar{\sigma}_m^{0(k)} \tag{5}$$

where $\bar{S}_{ij}$ is the in-plane compliance matrix.

Consider a $[\pm\theta_m/90_n]_s$ laminate with a finite gauge length of $2l$ and width of w with transverse ply cracks. The potential energy is written as:

$$PE = U - 2(h_1+h_2)w2l\bar{\sigma}_i\bar{\varepsilon}_i \tag{6}$$

where U is the total strain energy of the laminate. Using the constitutive relation defined in Eq. (2), the total strain energy is found

$$U = \frac{1}{2}\sum_{k=1}^{2} 2lw2h_k Q_{ij}^k(\bar{\varepsilon}_i + \bar{\varepsilon}_i^{0(k)})(\bar{\varepsilon}_j + \bar{\varepsilon}_j^{0(k)}) \tag{7}$$

The energy release rate is defined as the first partial derivative of the potential energy corresponding to the crack surface area, A, with a fixed applied laminate stresses

$$G = -\left(\frac{\partial PE}{\partial A}\right)\Big|_{\bar{\sigma}_i} \tag{8}$$

Rearranging Eqs. (4)–(7) and substituting them into Eq. (8) gives the energy release rate associated with matrix cracking, which can be derived and expressed as [20]:

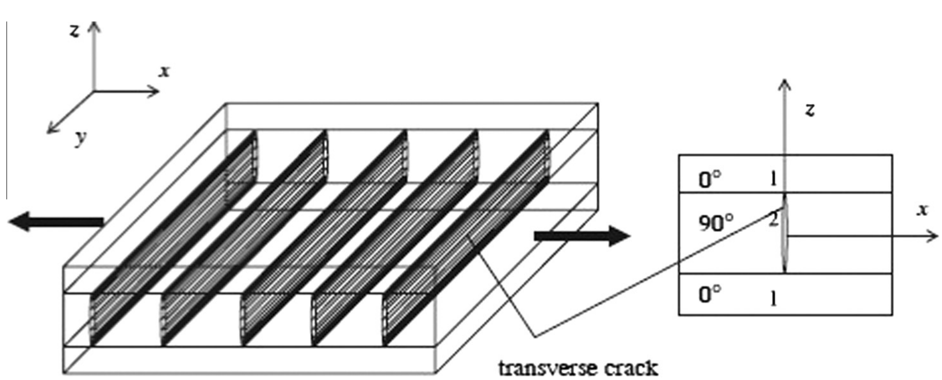

Fig. 1. Schematics of composite laminate with transverse matrix crack [4].