Applied Mathematical Modelling 000 (2018) 1–14

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Thermal vibration analysis of functionally graded shallow spherical caps by introducing a decoupling analytical approach

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ARTICLE INFO

Article history:
Received 4 May 2017
Revised 8 February 2018
Accepted 13 February 2018
Available online xxx

Keywords:
Analytical approach
Free vibration
Shallow spherical cap
Functionally graded materials

ABSTRACT

Due to many applications of spherical shells on a circular planform such as the nose of the plane and spacecraft and caps of pressurized cylindrical tanks, in this article, free vibration analysis of a thin functionally graded shallow spherical cap under a thermal load is considered. A decoupling technique is employed to analytically solve the equations of motion. Introducing some new auxiliary and potential functions as well as using the separation method of variables, the governing equations of the vibrated functionally graded shallow spherical cap were exactly solved. The superiority of the relations is validated by some comparative studies for various types of boundary conditions. Also, thermal buckling phenomenon is considered. Using new different material models, efficiency of the functionally graded materials is investigated when the shell is subjected to a temperature gradient. The effects of various parameters such as radius of curvature, material grading index and thermal gradient are discussed.

1. Introduction

Shallow spherical shells are a kind of doubly curved shells with two equal curvatures. The geometrical property of the spherical shells leads to especial applications such as various reservoirs, interstages and caps. Also, in some of these applications, spherical shells confront a high level of thermal gradient. For example, when a shallow spherical shell with a circular planform is used in the nose of a spacecraft, it was exposed to exceedingly high temperatures while in orbit and especially during the searing 1650 °C heat of atmospheric reentry. Due to high thermal resistance with low thermal stresses, functionally graded materials are a good choice to employ in these structures. Therefore, it is very important to have an accurate procedure for the free vibration analysis of FG shallow spherical shells with a circular planform as spherical caps when they encounter with a thermal load.

Many researches have been carried out on the vibration behavior of spherical shell with various materials and using different solution methods. Wu and Heyliger [1] analyzed the free asymmetric and axisymmetric vibration of layered piezoelectric spherical shells using a two-dimensional first order shear deformable shell theory. A three-dimensional Ritz discrete-layer approximation method is used to solve the problem. Xu and Chia [2] investigated the nonlinear vibration of symmetrically laminated thin spherical caps with flexible supports by a multi-mode approach. Lee [3] applied the pseudospectral method to the axisymmetric and asymmetric free vibration analysis of spherical caps. The displacements and

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https://doi.org/10.1016/j.apm.2018.02.018
0307-904X/

Please cite this article as: A.R. Barzegar, M. Fadaee, Thermal vibration analysis of functionally graded shallow spherical caps by introducing a decoupling analytical approach, Applied Mathematical Modelling (2018), https://doi.org/10.1016/j.apm.2018.02.018

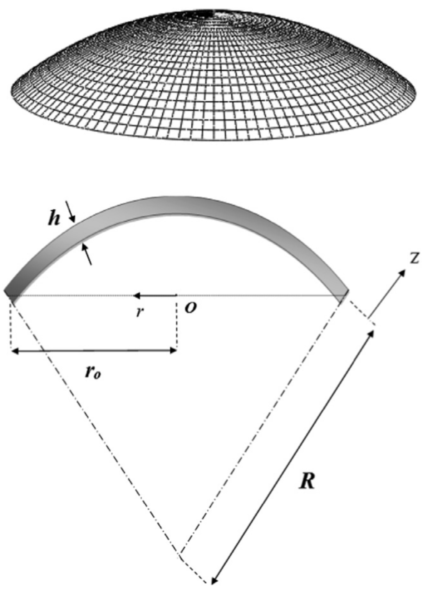

Fig. 1. Geometry of a FG spherical cap.

the rotations are presented by Chebyshev polynomials and Fourier series. Ganapathi [4] studied the dynamic stability behavior of a clamped functionally graded spherical shell subjected to external pressure load. The governing equations are solved employing the Newmark's integration technique coupled with a modified Newton–Raphson iteration scheme. Ye et al. [5] considered free vibration of laminated functionally graded spherical shells with general boundary conditions. The study is based on the three-dimensional shell theory of elasticity and the energy based Rayleigh–Ritz procedure. Okhovat and Boström [6] derived dynamic equations for an isotropic spherical shell using a series expansion technique. By using the expansions of the displacement components, the three-dimensional elastodynamic equations yield a set of recursion relations among the expansion functions. Efraim and Eisenberger [7] presented a dynamic stiffness method for obtaining the free vibration frequencies and mode shapes of thick spherical shell segments with variable thickness and different boundary conditions. The solution is obtained using the exact element method algorithm. Birman et al. [8] introduced a theoretical formulation for spherical shells reinforced by meridional and circumferential stiffeners. The analysis employed the Donnell–Mushtari–Vlasov version of Love's theory of shells together with a smeared stiffeners technique. Using Generalized Differential Quadrature (GDQ) Method, Tornabene and Viola [9] studied the dynamic behavior of spherical shell panels using the FSDT. Ferreira et al. [10] used a meshless method based on the FSDT of Donnell to obtain the natural frequencies of cross-ply composite spherical shells. Zenkour [11] discussed on static and dynamic responses of anisotropic spherical shells under a uniformly distributed transverse load. Analytical solutions using the mixed variational formulation are presented to solve the problem for various boundary conditions. Liew et al. [12] presented the elasticity solutions for free vibration analysis of spherical shell panels of rectangular planform. The problem has been solved via a three-dimensional displacement-based energy formulation as the *p*-Ritz method. Kang et al. [13] carried out three-dimensional analysis of free vibration of spherical shell segments with variable thickness. Potential (strain) and kinetic energies of the spherical shell segment are formulated, and upper bound values of the frequencies are obtained by minimizing the frequencies.

After a comprehensive literature review by the authors, it has been seen that in most papers related to the free vibration analysis of shallow spherical shells, numerical methods were used to solve the problem. Also, thermal gradient effects on the natural frequencies of these structures were not analytically analyzed. Hence, in this paper, thin shallow shell theory in polar coordinates is employed to obtain the dynamics equations of a spherical FG cap under a thermal gradient and then, using a decoupling procedure, exact closed form solution of the problem is obtained. This new exact analytical approach can be applied for any boundary condition without any usage of approximate methods. To show the accuracy of the results, the natural frequencies are compared with the available data in literature and a finite element method (FEM) analysis. Finally, parametric study has been carried out to achieve various effects on the natural frequencies and thermal buckling load.

2. Formulations

2.1. Geometric interpretation

Fig. 1 depicts a thin shallow spherical cap with radius of curvature R and uniform thickness h. Due to the shallowness of the shell as well as having a circular planform with radius r_0, a polar coordinate system (r, θ) is employed to formulate the problem.

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