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Evolving chaos: Identifying new attractors of the generalised Lorenz family

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ABSTRACT

In a recent paper, we presented an intelligent evolutionary search technique through genetic programming (GP) for finding new analytical expressions of nonlinear dynamical systems, similar to the classical Lorenz attractor's which also exhibit chaotic behaviour in the phase space. In this paper, we extend our previous finding to explore yet another gallery of new chaotic attractors which are derived from the original Lorenz system of equations. Compared to the previous exploration with sinusoidal type transcendental nonlinearity, here we focus on only cross-product and higher-power type nonlinearities in the three state equations. We here report over 150 different structures of chaotic attractors along with their one set of parameter values, phase space dynamics and the Largest Lyapunov Exponents (LLE). The expressions of these new Lorenz-like nonlinear dynamical systems have been automatically evolved through multi-gene genetic programming (MGGP). In the past two decades, there have been many claims of designing new chaotic attractors as an incremental extension of the Lorenz family. We provide here a large family of chaotic systems whose structure closely resemble the original Lorenz system but with drastically different phase space dynamics. This advances the state of the art knowledge of discovering new chaotic systems which can find application in many real-world problems. This work may also find its archival value in future in the domain of new chaotic system discovery.

1. Introduction

Investigation of new chaotic attractors showing rich phase space dynamics has been widely researched in many studies particularly in the field of cryptography and secure communication [1] and explaining naturally occurring complex systems in biology, economics, chemistry and physics [2]. There have been several claims of inventing new chaotic attractors in recent years as a derivative of the celebrated Lorenz system [3,4] to the Lorenz family of systems [5,3] e.g. Lu et al. [6], Chen and Ueta [7], Lu and Chen [8], Liu et al. [9], Qi et al. [10], Sprott [11–14] etc. Given a highly complex time series, there have been very few successful results to find out the structure of the underlying chaotic system, especially when the order and exact functional complexity of the dynamical system is not precisely known. In most cases, this inverse problem is infeasible but given a pool of chaotic attractor structures, it is rather easy to simulate the state variables and compare it with the observation. Using the Takens' embedding theorem [15,16] and recorded time series of just one state variable

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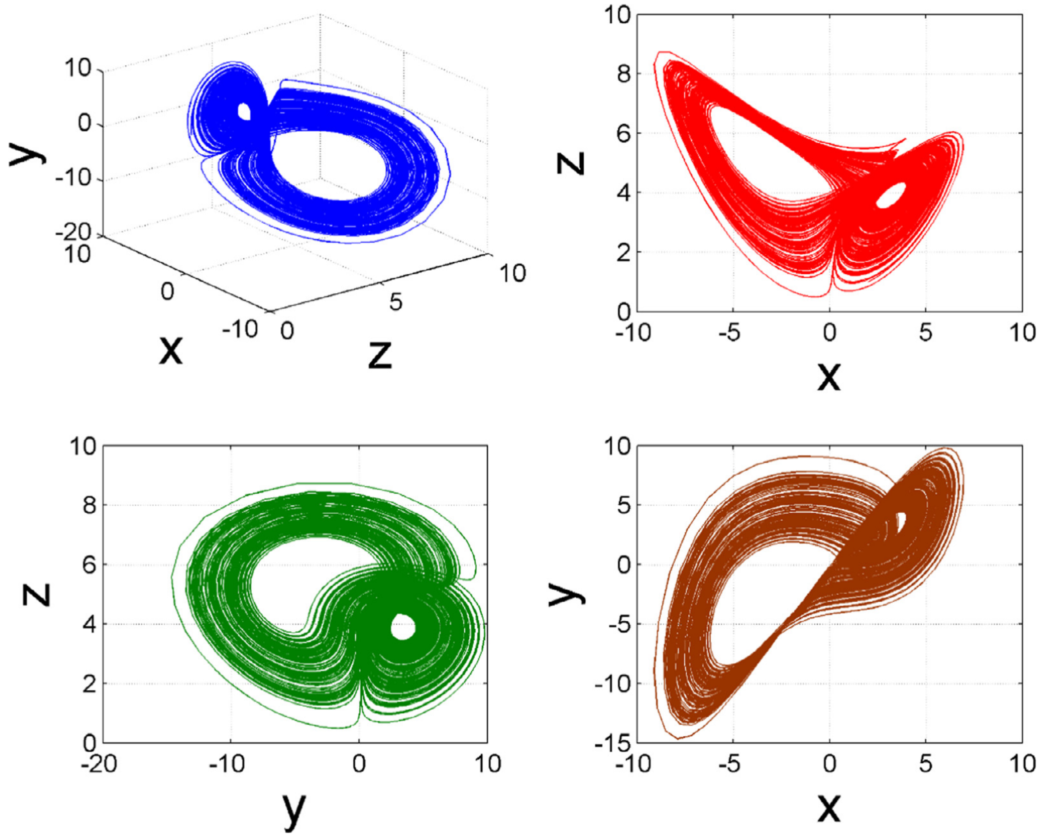

Fig. 1. Phase space dynamics of Lorenz-XY15 (uneven wings).

of finite length, it is possible to reconstruct the original phase space dynamics of the underlying attractor without precisely knowing its mathematical structure. Under such a scenario and with a proper choice of embedding delay, the LLE of the reconstructed attractors closely approach that of the original one.

Our previous study in [17] report more than 100 chaotic attractors having at least a single sinusoidal nonlinearity in one of the state equations. It was also argued in [17] that the rich phase space dynamics may be an effect of increased number of equilibrium points due to the transcendental terms e.g. sinusoidal function of the state variables. Here, we explore a different family of attractors having rather a much simpler cross-product and higher power type nonlinearity involving the three state variables. We here show that even with simple algebraic expressions without the previously explored transcendental terms e.g. sinh, cosh, sin, cos, exp, as shown in [17] and Sprott [11–14], quite complex phase space dynamics can be generated using an evolutionary search with genetic programming. During automatic evolution of chaotic system expressions, the state time series based time delay embedding method has been employed to calculate the Lyapunov exponent [16] for the initial screening. However, numerical calculation of the LLE based on finite length of only one state time series could lead to false discovery of many nonlinear dynamical systems as chaotic, if a positive LLE criteria were imposed on the values calculated on these set of simulations. Therefore, after the initial time series based evolution of chaotic systems and LLE computation with time delay embedding, the true LLEs have been recalculated using a symbolic differentiation scheme for each of the newly evolved expression of the state equation through genetic programming, while calculating the Jacobian matrix of these new nonlinear dynamical system. We believe that these new chaotic attractors are going to serve as a useful archival reference for future designers of chaotic cryptography and many natural scientists as these extends the generalised Lorenz family with cross-product nonlinearity to a much wider library of Lorenz like attractors.

2. Genetic programming to evolve new chaotic attractors

2.1. Search method and objective function

Single and multigene GP have been used to search for the chaotic system expressions as reported in our previous exploration [17]. The single gene GP helps evolving a single state equation while keeping the other two as that of the Lorenz attractor, whereas the multigene GP simultaneously evolves two or all the three state equations to find a completely new chaotic system compared to the structures of the classical Lorenz system [3]. The single or multi-gene GP evolves with