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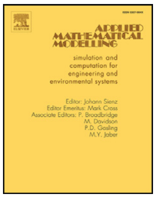

Variable-sampling plans based on lifetime-performance index under exponential distribution with censoring and its extensions

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ABSTRACT

The exponential distribution is commonly used to model electronics components and systems, mechanical fatigue failures, and some corrosion processes that usually do not wear out until long after the product's expected life span. Herein, based on a lifetime-performance index, we design acceptance-sampling plans for an exponential population with and without censoring using statistical and decision-theoretic methodologies that minimize the number of failures required during inspection. Moreover, the performance of established sampling plans is compared with that of the recently proposed approximation approach with full-ordered observed exponential data. We also investigate the industrial applicability of our recommended sampling plans in a case study. To encompass more real applications, the extension of the methodologies to the two-parameter Weibull distribution is also included.

1. Introduction

Manufacturers need quality (reliability) information before releasing a product. Potential customers require this information before purchasing a product. Thus, quality inspection and reliability testing are necessary to demonstrate and assure product quality and reliability [1]. Acceptance-sampling plans have been widely applied for both purposes. Such plans assist in determining an optimal decision based on economic considerations, thereby guaranteeing a quality (reliability)-related quantity of interest (called a process parameter) that meets or exceeds a specified requirement at a desired level of confidence.

In this paper we are dealing with product's lifetime-performance evaluation. Some phrases and words will be used interchangeably, so we must mention here to avoid further comprehension problems and redundancies. Product lifetimes are both reliability and quality characteristics; thus, "reliability" and "quality" are not distinguished. Furthermore, lifetime data are obtained through lifetime testing that could equally well be called reliability testing in the reliability field or quality inspection in the quality field; thus, "lifetime test," "reliability test," and "quality inspection" are equivalent. In addition, when dealing with the topic of acceptance-sampling plans, "accepting or rejecting the population" is jargon in reliability testing [2], whereas "accepting or rejecting the batch (lot)" is widely used in quality inspection; thus "population," "batch," and "lot" are interchangeable [3]. The above statements are also implicitly emphasized in [1].

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Quantification of the location and dispersion parameters is central to understanding the quality of a process. In practice, the process mean and standard deviation are unknown; thus, a random sample is required to estimate the unknown parameters. However, determining the correct sample size can be critical because the tests are usually expensive and obtaining prototypes is often difficult. If the sample size used is too small, little information can be obtained from a test, limiting ones' ability to draw meaningful conclusions; however, if the sample is too large, information obtained through the test might be beyond what is required, thereby incurring unnecessary costs [4].

Traditionally, there are two methods for determining the correct sample size required in a reliability test: an estimation approach of unknown process parameters based on a confidence interval [5] and a risk-control approach for hypothetical process parameters based on controlling Type I and Type II errors [6]. Practically, however, the estimators of the unknown process parameters used in either case are not unitless and sometimes are not convenient summary statistics in a plant or supply base for which various characteristics with disparate metric measures are considered [7]. Process capability indices are dimensionless quantities that measure the relationships between quality-characteristic specification limits and actual performance of an in-control process [8-10]. These indices have played an integral role in continuously improving product quality and reliability.

However, most indices that have been applied thus far assume that quality-characteristic measurements are normally distributed [11-15]. Generally, product lifetime T with only positive values is the-larger-the-better type of quality characteristic. It inherits the properties of an exponential, gamma, or Weibull distribution [16]. The exponential distribution is widely used to model electronic components and systems, mechanical fatigue failures, and some corrosion processes that usually do not wear out until long after the product's expected life span in which they are installed [17]. It is also considered as an excellent model for the relatively stable period of low failure risk, which characterizes the bathtub curve's middle portion. This phase corresponds to the product's useful life and is known as the curve's intrinsic failure portion [6].

Currently, many devices using advanced manufacturing technologies have high quality and a long lifetime T. Collecting complete item lifetimes for a life test is time-consuming and expensive. Consequently, practitioners create acceptance-sampling plans for life testing with failure-censoring schemes and make a proper decision regarding lot sentences as soon as cumulative life information is sufficient [18]. Herein, we consider Type II right censoring (failure-censoring scheme), referring to a situation in which only the s smallest lifetimes $t_{(1)} \le \ldots \le t_{(s)}$ in a random sample of n are observed; here, s is a specified integer between 1 and n. This censoring scheme arises when n individuals begin a study at the same time, with the study terminating once s lifetimes (failures) have been observed [19].

The lifetime-performance index C_L, a type of process capability index developed by [7,20], measures the performance of a process with a nonnegative lifetime characteristic T for which a lower specification limit L has been set. Several studies have focused on the parameter-estimation and hypothesis-testing approaches for the unknown C_L index based on confidence intervals [21-23]; however, examination has been limited to a unilateral viewpoint of risk either from the producer or consumer.

Recently, Aslam et al. [24] used an approximation approach for remedying data to develop C_L-similar sampling plans for an exponential population. Their idea follows the research of Johnson and Kotz [25] and Nelsons [26], in which a power function is first used to transform exponentially distributed lifetime data into a Weibull distribution and then the best power form is selected to approximate the Weibull distribution with a normal distribution. A similar technique was applied recently to control charts for monitoring the quality characteristic with an exponential distribution [27,28]. However, the approximation approach is limited because it does not address the theoretical and practical interest problems, e.g., estimator properties, estimator's exact sampling distribution, and lifetime data with censoring information.

Therefore, based on the lifetime-performance index, we design acceptance-sampling plans for an exponential population with and without censoring using statistical estimation and theoretical decision-making methodologies that minimize the number of failures required for an inspection. The performance of established sampling plans is also compared to a recently proposed approximation approach [24] with full-ordered observed exponential data. The rest of this paper is organized as follows. Section 2 briefly introduces C_L and emphasizes the advantages of using it. The estimation of C_L for exponentially distributed product lifetime with Type II censoring is also provided. Section 3 incorporates C_L into sampling plans for submitted lots with Type II censoring. The plan parameters are determined using an optimization model with nonlinear constraints. Moreover, decision criteria's determination is analyzed and discussed. Section 4 compares our proposed exact sampling plans to a recently published approximation approach with full-ordered observed exponential data. Section 5 presents an example illustrating the proposed sampling plan's application. Section 6 extends the sampling plans to product lifetime with a two-parameter Weibull distribution. Conclusions are drawn in Section 7.

2. Lifetime-performance index for exponentially distributed lifetime data

The index C_L was developed to provide a dimensionless quantity for measuring the performance of a process with a nonnegative lifetime characteristic T that allows only a lower lifetime limit L. It is defined as (see [20])

$$C_L = \frac{\mu_T - L}{\sigma_T}, \tag{1}$$

where μ_T and σ_T are the mean and standard deviation of the lifetimes, respectively.