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# A constrained variational problem arising in attractive Bose-Einstein condensate with ellipse-shaped potential

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**Abstract**

We consider a minimization problem for the variational functional associated with a Gross-Pitaevskii equation arising in the study of an attractive Bose-Einstein condensate. Under an ellipse-shaped trapping potential, that is, the bottom of the trapping potential is an ellipse, we prove that any minimizer of the minimization problem blows up at one of the endpoints of the major axis of the ellipse if the parameter associated to the attractive interaction strength approaches a critical value.

*Keywords:* Variational method; minimization problem; elliptic equation; energy estimates; ellipse-shaped potential.
*2010 MSC:* 35J20; 35J60; 47J10

**Declarations of interest:** none.

## 1. Introduction

In this paper, we are concerned with the following constrained minimization problem

$$e_\beta := \inf\left\{E_\beta(u) : u \in \mathcal{H} \text{ and } \int_{\mathbb{R}^2} u^2 dx = 1\right\}, \tag{1.1}$$

where $\mathcal{H} = \left\{u \in H^1(\mathbb{R}^2) : \int_{\mathbb{R}^2} V(x)u^2 dx < +\infty\right\}$, and

$$E_\beta(u) = \int_{\mathbb{R}^2} (|\nabla u|^2 + V(x)u^2)dx - \frac{\beta}{2}\int_{\mathbb{R}^2} |u|^4 dx, \tag{1.2}$$

and $\beta > 0$, $V(x) \geq 0$ is a trapping potential. This problem is related to the study of ground states for a Gross-Pitaevskii (GP) equation arising in attractive Bose-Einstein condensate (BEC), the parameter $\beta > 0$ means that BEC has attractive interactions and $\beta$ is related to the interaction strength, see e.g., [3] and the references therein for more physical background. If $V(x)$ is a trapping potential such that

$$V(x) \in L^\infty_{loc}(\mathbb{R}^2, \mathbb{R}^+), \ \lim_{|x|\to\infty} V(x) = \infty \text{ and } \inf_{x\in\mathbb{R}^2} V(x) = 0, \tag{1.3}$$

it was proved in [1, 5, 13] that the minimization problem (1.1) has a minimizer if and only if

$$\beta \in [0, \beta_*) \text{ and } \beta_* = \|Q\|^2_{L^2},$$

where $Q(x)$ is the unique (up to translations) radially symmetric positive solution of the equation

$$-\Delta u + u = u^3, \ u \in H^1(\mathbb{R}^2). \tag{1.4}$$

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