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Relaxation oscillations in a slow-fast modified Leslie-Gower model

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Abstract

In this paper, we prove the existence and uniqueness of relaxation oscillation cycle of a slow-fast modified Leslie-Gower model via the entry-exit function and geometric singular perturbation theory. Numerical simulations are also carried out to illustrate our theoretical result.

Keywords: Slow-fast system, entry-exit function, geometric singular perturbation, relaxation oscillation.

1. Introduction

Alaoui and Okiye [3] in 2003 proposed the next modified Leslie-Gower model:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}T} &= \left(r_1 - b_1 x - \frac{a_1 y}{x+k_1}\right)x,\\ \frac{\mathrm{d}y}{\mathrm{d}T} &= \left(r_2 - \frac{a_2 y}{x+k_2}\right)y,\end{aligned} \tag{1.1}$$

where x represents the prey density and y the predator density, and the parameters r_1, b_1, a_1, k_1, r_2, a_2, k_2 are positive and have biological meaning described as follows. The parameters r_1 and b_1 represent the intrinsic growth rate and the strength of competition among individuals for the prey, respectively. The natural growth rate of the predator is given by r_2, while a_2 and a_1 are respectively the maximum values of the per capita reduction rates of the predators and prey species. The parameters k_1 and k_2 measure the extent to which environment provides protection to prey x and predator y, respectively.

System (1.1) has been investigated by several researchers in different aspects. Alaoui and Okiye [3] studied the boundedness of solutions and global stability of the positive equilibrium points of the system. By utilizing the coincidence degree theorem and Lyapunov function, Zhu and Wang [20] obtained some sufficient conditions for the existence and global attractivity of positive periodic solutions of the system.

Now consider the rescaling

$$x = \frac{r_1}{b_1}\bar{x}, \quad y = \frac{r_1 r_2}{a_2 b_1}\bar{y}, \quad T = \frac{1}{r_1}t. \tag{1.2}$$

Substitute (1.2) into system (1.1) and still use x and y to denote $\bar{x}$ and $\bar{y}$, it yields

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= x' = x(1-x) - \frac{axy}{x+e_1} := f(x,y,\lambda),\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= y' = \varepsilon y\left(1 - \frac{y}{x+e_2}\right) := \varepsilon g(x,y,\lambda),\end{aligned} \tag{1.3}$$

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