Accepted Manuscript

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PII: S0893-9659(18)30004-1
DOI: https://doi.org/10.1016/j.aml.2018.01.002
Reference: AML 5413

To appear in: *Applied Mathematics Letters*

Received date : 22 November 2017
Revised date : 2 January 2018
Accepted date : 2 January 2018

Please cite this article as: S.-H. Park, M.J. Lee, J.-R. Kang, Blow-up results for viscoelastic wave equations with weak damping, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.01.002

BLOW-UP RESULTS FOR VISCOELASTIC WAVE EQUATIONS WITH WEAK DAMPING

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Abstract. In this work we consider a viscoelastic wave equation of the form

$$u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds + h(u_t) = |u|^{p-2}u$$

with Dirichlet boundary condition. There are much literature on the blow-up result of solutions for the wave equation with damping term having polynomial growth near zero. However, to my knowledge, there is no blow-up result of solutions for the viscoelastic wave equation without polynomial growth near zero assumption on the damping term. This work is devoted to study a finite time blow-up result of solution with nonpositive initial energy as well as positive initial energy without imposing any restrictive growth near zero assumption on the damping term.

1. Introduction

In this paper we investigate a blow-up result for the following viscoelastic wave equation

$$u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(s)ds + h(u_t) = |u|^{p-2}u \text{ in } \Omega \times \mathbb{R}^+, \tag{1.1}$$

$$u = 0 \text{ on } \partial\Omega \times \mathbb{R}^+, \tag{1.2}$$

$$u(0) = u_0, \ \ u_t(0) = u_1 \quad \text{on } \Omega, \tag{1.3}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with sufficiently smooth boundary $\partial\Omega$.

In the absence of the viscoelastic term ($g = 0$), the problem has been extensively studied and results concerning existence and nonexistence have been established. For $h = 0$, the source term $|u|^{p-2}u$ causes finite time blow up of solutions with negative initial energy ([1]). When the linear damping case ($h(u_t) = au_t$), the interaction between the damping and the source terms was first considered by Levine ([2,3]). He proved that solutions with negative initial energy blow up in finite time. Georgiev and Todorova [4] extended Levin's result to the nonlinear damping case ($h(u_t) = au_t|u_t|^{m-2}, m > 2$). They showed that solutions blow up in finite time when the initial energy is sufficiently negative and $p > m$. Messaoudi [5] extended the blow up result of [4] to solutions with negative initial energy only. Concerning the local and global existence for the solutions of the wave equation with nonlinear boundary damping and source terms, we refer to [6,7]. Ha [8] investigated the blow up for semilinear wave equation with boundary damping and source terms. Recently, Ha [9] generalized the result of [8] under a weaker assumption, that the boundary damping term did not necessarily have polynomial growth near zero.

2010 Mathematics Subject Classification : 35L70, 35B41.
keywords : blow-up, wave equation, viscoelasticity.
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