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New benchmark solutions for free vibration of clamped rectangular thick plates and their variants

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ABSTRACT

It is of significance to explore benchmark analytic free vibration solutions of rectangular thick plates without two parallel simply supported edges, because the classic analytic methods are usually invalid for the problems of this category. The main challenge is to find the solutions meeting both the governing higher order partial differential equations (PDEs) and boundary conditions of the plates, i.e., to analytically solve associated complex boundary value problems of PDEs. In this letter, we extend a novel symplectic superposition method to the free vibration problems of clamped rectangular thick plates, with the analytic frequency solutions obtained by a brief set of equations. It is found that the analytic solutions of clamped plates can simply reduce to their variants with any combinations of clamped and simply supported edges via an easy relaxation of boundary conditions. The new results yielded in this letter are not only useful for rapid design of thick plate structures but also provide reliable benchmarks for checking the validity of other new solution methods.

1. Introduction

The plate structures have found broad applications in mechanical, civil, aerospace and ocean engineering. The free vibration problem has been a key issue for plates because it represents one of the main mechanical behaviors of these structures. The analysis for plates is thus of particular importance for understanding the performances and further guiding the designs of related engineered devices and structures. Depending on the ratio of thickness to minimum plane dimension, the plates are often classified into two categories, i.e., thin and thick plates. Accordingly, the widely adopted mechanics models are classified into the Kirchhoff theory-based thin plate model and more complex Mindlin/Reissner theory-based thick plate model [1,2] or the other higher-order plate models [3,4].

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From the mathematical point of view, to solve the plate problems is to seek the solutions meeting both the governing higher order partial differential equations (PDEs) and boundary conditions of the plates, i.e., to solve typical complex boundary value problems of PDEs; thus, developing effective mathematical methods becomes crucial. For rectangular plates that are commonly encountered in engineering practice, the classic semi-inverse method has been applied a long time ago, but the analytic solutions stay on the plates with two parallel simply supported edges due to invalidity of the method for the other cases. Therefore, many numerical methods open the possibilities of solving the plates having various boundary conditions with satisfactory accuracy. The conventional numerical methods, such as the finite element method (FEM) and finite difference method (FDM), have been summarized in Leissa's famous monograph on vibration of plates [5].

In recently years, some new advanced numerical methods have been explored for plates' vibration problems. For example, Civalek et al. [6–9] developed the discrete singular convolution (DSC) method for the free vibration analysis of both thin and thick plates; the method is computationally efficient and accurate, and very good convergence is achieved. Kumari and Behera [10] used the extended Kantorovich method to obtain the free vibration solutions for Lévy-type rectangular laminated plates; the solutions can serve as a benchmark to assess the other two-dimensional theories and three-dimensional numerical solutions. Malekzadeh and Karami [11] presented a differential quadrature (DQ) procedure for free vibration analysis of moderately thick plates with variable thickness; only few grid points can yield accurate results for higher-order modes, demonstrating the advantage of low computation cost of the method. Cho et al. [12] applied the assumed mode and mode superposition method to develop a very simple, fast, and accurate procedure for the forced vibration of plates and stiffened panels, which is especially appropriate for early design stage. In addition, the Rayleigh–Ritz method is still popular in handling the plate vibration problems; the newly adopted admissible functions include the Chebyshev polynomials [13], new trigonometric series [14,15], etc. Compared to the numerical solutions, the analytic solutions for plates are much fewer due to the mathematical difficulty. Representative analytic methods include the semi-inverse superposition method [16], finite integral transform method [17], symplectic method [18], etc. The applications of these approaches have been focused on thin plates.

In this letter, we report the analytic free vibration solutions of clamped rectangular thick plates by extending an up-to-date symplectic superposition method, which combines the rationality of the symplectic elasticity approach, pioneered by Yao et al. [19] and further advanced by Lim et al. [18,20,21] and Li et al. [22–25] for plate problems, and generality of the superposition method, such that the focused problems could be solved in a rigorous way, without any predetermined admissible solutions. To save space, construction of the new governing equation in the matrix form is omitted in this letter. The interested reader is referred to a recent study of ours [25] for details. After briefly introducing the solution procedure of the symplectic superposition method, this work mainly demonstrates the application to clamped rectangular thick plates and their variants, i.e., those with any combinations of clamped and simply supported edges. Accordingly, the benchmark analytic solutions will be the main outcomes.

2. Symplectic superposition method-based analytic solutions for clamped thick plates and their variants

The governing equations for free vibration of a thick plate in the rectangular coordinate system xoy (Fig. 1(a)) based on the Mindlin theory are formulated in the symplectic space as [25]

$$\partial \mathbf{Z}/\partial y = \mathbf{H}\mathbf{Z} \tag{1}$$

Here $\mathbf{Z} = [\Phi, w, \Psi, \alpha, \beta, \theta]^{\mathrm{T}}$ is the state vector, whose components include the modal displacement w and five functions defined by $\Phi = \partial\psi_x/\partial x + \partial\psi_y/\partial y$, $\Psi = \partial\psi_y/\partial x - \partial\psi_x/\partial y$, $\alpha = D/\left(\rho h\omega^2\right)\partial\Phi/\partial y$, $\beta = \partial w/\partial y$ and $\theta = \partial\Psi/\partial y$, respectively. ψ_x and ψ_y are the rotating angles about the y- and x-