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Novel differential quadrature element method for vibration analysis of hybrid nonlocal Euler–Bernoulli beams

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ABSTRACT

A novel differential quadrature element method is presented for free vibration analysis of hybrid nonlocal Euler–Bernoulli beams with any combination of boundary conditions. Explicit formulas of computing the weighting coefficients of various derivatives are derived. Sixth-order differential equations are successfully solved by using the proposed method. Accurate frequencies are obtained and presented. The proposed method can be also used for accurate solutions of various sixth-order differential equations and beam structures with minimum computational effort.

1. Introduction

Currently the behavior of free vibration of micro/nano-sized structures is interested by many researchers [1]. The widely used nonlocal continuum theories are the Eringen's nonlocal elasticity theory [2,3] and the strain gradient elasticity theory [4]. To model the micro/nano-structures more accurately, various hybrid nonlocal Euler–Bernoulli beam models have been proposed by Lim et al. [5]. Since the models possess two or more independent small length-scale parameters and thus are more flexible in accurately modeling the micro/nano-sized structures. The Eringen's nonlocal elasticity theory [2,3] and the strain gradient elasticity theory [4] can be regarded as the special cases of the hybrid nonlocal Euler–Bernoulli beam models.

Both the strain gradient theory and the hybrid nonlocal Euler–Bernoulli beam model with two independent small length-scale parameters result in a sixth-order differential equation. Perhaps due to the difficulty in solving the sixth-order differential equation with general boundary conditions, very few works on the hybrid nonlocal Euler–Bernoulli beams have been reported thus far [1]. Analytical solutions may be only possible for the simple combinations of boundary condition. Therefore, numerical methods must be resorted to for solutions.

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Besides the well-known finite element method (FEM) [2,3], several efficient numerical methods are available in literature, such as the discrete singular convolution [6,7], the quadrature element method [8,9], the differential quadrature method [10,11] and the differential quadrature element method (DQEM) [11,12], however, none of them in its current version can be used to solve the sixth-order differential equation with different combinations of boundary conditions.

The present work is the first attempt to obtain vibration solutions of hybrid nonlocal Euler–Bernoulli beams under different combinations of boundary conditions. A novel DQEM is proposed and explicit formulas of computing the weighting coefficients are given. The rate of convergence of the DQEM is studied. The proposed method can be used for solutions of various sixth-order differential equations with any combination of boundary conditions.

2. Hybrid nonlocal Euler–Bernoulli beam

Denote E, I, A, ρ and w as Young's modulus, the second moment of the cross-sectional area, the cross sectional area, the mass density, and the transverse displacement. For simplicity, the kth order derivative with respect to x is denoted by superscripts (k) through out the paper. For the free vibration analysis, the governing equation of the hybrid nonlocal Euler–Bernoulli beam is given by [5]

$$EIw^{(4)} - l^2 EIw^{(6)} = \rho A\omega^2 w - (ea)^2 \rho A\omega^2 w^{(2)} \tag{1}$$

where symbols l and ea represent the two independent small length-scale parameters, and ω is the circular frequency.

The shear force, bending moment and high-order bending moment are defined as [5]

$$Q_x = EI\left(w^{(3)} - l^2 w^{(5)}\right) + \rho A\omega^2 (ea)^2 w^{(1)} \tag{2}$$

$$M_x = EI\left(w^{(2)} - l^2 w^{(4)}\right) \tag{3}$$

$$M_{xx} = -l^2 EIw^{(3)} \tag{4}$$

The beam has twenty-one combinations of boundary conditions. Only three of them, denoted by S_a-S_a, C_b-C_b, and C_a-F_a, are considered. They are: (a) S_a-S_a: $w = M_x = w^{(2)} = 0(x = 0, L)$; (b) C_b-C_b: $w = w^{(1)} = M_{xx} = 0(x = 0, L)$; and (c) C_a-F_a: $w = w^{(1)} = w^{(2)} = 0(x = 0)$ and $Q_x = M_x = w^{(2)} = 0(x = L)$.

3. DQEM and solution procedures

Let N be the number of nodal points of the differential quadrature (DQ) beam element, and $x_i (i = 1, 2, \ldots, N)$ the nodal coordinates in local coordinate system. Similar to the existing DQEM [11,12], the stiffness equation of the element is directly formulated by using the differential equation and generalized forces defined by Eqs. (1)–(4).

The transverse displacement $w(x)$ within the DQ beam element is assumed as

$$w(x) = \sum_{j=1}^{N} \varphi_j(x) w_j + \psi_1(x) w_1^{(1)} + \psi_N(x) w_N^{(1)} + \Gamma_1(x) w_1^{(2)} + \Gamma_N(x) w_N^{(2)} = \sum_{j=1}^{N+4} H_j(x)\delta_j \tag{5}$$

where the shape function $H_j(x)$ is the Hermite-type function and its order is $(N+3)$, $\delta_j = w_j$ $(j = 1, 2, \ldots, N)$, $\delta_{N+1} = w_1^{(1)}, \delta_{N+2} = w_N^{(1)}, \delta_{N+3} = w_1^{(2)}, \delta_{N+4} = w_N^{(2)}, H_{N+1} = \psi_1$, $H_{N+2} = \psi_N, H_{N+3} = \Gamma_1, H_{N+4} = \Gamma_N$, and $H_j = \varphi_j$ $(j = 1, 2, \ldots, N)$, respectively.