Accepted Manuscript

Infinitely many nodal solutions for semilinear Robin problems with an indefinite linear part

Nikolaos S. Papageorgiou, Vicenţiu D. Rădulescu

PII: S0893-9659(16)30246-4
DOI: http://dx.doi.org/10.1016/j.aml.2016.08.011
Reference: AML 5075

To appear in: *Applied Mathematics Letters*

Received date: 14 July 2016
Revised date: 17 August 2016
Accepted date: 17 August 2016

INFINITELY MANY NODAL SOLUTIONS FOR SEMILINEAR ROBIN PROBLEMS WITH AN INDEFINITE LINEAR PART

NIKOLAOS S. PAPAGEORGIOU AND VICENŢIU D. RĂDULESCU

Abstract. We consider a semilinear Robin problem driven by the Laplacian plus an indefinite potential and with a Carathéodory reaction $f(z,x)$ with no growth restriction on the x-variabile. We only assume that $f(z,\cdot)$ is odd and superlinear near zero. Using a variant of the symmetric mountain pass theorem, we show that the problem has a whole sequence of distinct smooth nodal solutions converging to the trivial one.

1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a C^2-boundary $\partial\Omega$. In this paper, we study the following semilinear Robin problem:

$$(1) \qquad \left\{ \begin{array}{l} -\Delta u(z) + \xi(z)u(z) = f(z,u(z)) \text{ in } \Omega, \\ \dfrac{\partial u}{\partial n} + \beta(z)u = 0 \text{ in } \partial\Omega\,. \end{array} \right\}$$

In this problem, $\xi \in L^s(\Omega)$ $(s > N)$ is an indefinite (that is, sign-changing) potential function and the reaction term $f(z,x)$ is a Carathéodory function (that is, for all $x \in \mathbb{R}$, $z \mapsto f(z,x)$ is measurable and for almost all $z \in \Omega$, $x \mapsto f(z,x)$ is continuous). No global growth condition is imposed on $f(z,\cdot)$, which can be arbitrary near $\pm\infty$. The only conditions on $f(z,\cdot)$ concern its behaviour near zero and we require that it is superlinear there. In the boundary condition, $\dfrac{\partial u}{\partial n}$ is the usual normal derivative defined by extension of the linear map

$$u \mapsto \frac{\partial u}{\partial n} = (Du, n)_{\mathbb{R}^N} \text{ for all } u \in C^1(\overline{\Omega}),$$

with $n(\cdot)$ being the outward unit normal on $\partial\Omega$. The boundary coefficient $\beta(\cdot)$ belongs to $W^{1,\infty}(\partial\Omega)$ and we assume that $\beta(z) \geqslant 0$ for all $z \in \partial\Omega$.

We are looking for nodal (that is, sign-changing) solutions of problem (1). Using an abstract multiplicity result of Heinz [3], Wang [14] and Kajikiya [4], we show that problem (1) admits a whole sequence $\{u_n\}_{n\geqslant 1} \subseteq H^1(\Omega)$ of distinct nodal solutions such that

$$u_n \in C^1(\overline{\Omega}) \text{ for all } n \in \mathbb{N} \text{ and } u_n \to 0 \text{ in } C^1(\overline{\Omega}).$$

Recently multiplicity results for semilinear elliptic problems with indefinite linear part, were proved by Castro, Cossio and Vélez [1], Papageorgiou and Papalini [5], Qin, Tang and Tang [12], Wu and An [15], Zhang and Liu [16], Zhang, Tang and Zhang [17] (Dirichlet problems), Papageorgiou and Rădulescu [6, 8] (Neumann problems) and Papageorgiou and Rădulescu [9] (Robin problems). None of the aforementioned works produces a whole sequence of nodal solutions and all impose a subcritical growth condition on the reaction term $f(z,\cdot)$. We mention also the very recent work of Papageorgiou and Rădulescu [10], which deals with nonlinear nonhomogeneous Robin problems with no potential term (that is, $\xi \equiv 0$) and a reaction term $f(z,x)$ of arbitrary growth in $x \in \mathbb{R}$. The authors of [10] produce a sequence of nodal solutions but under more restrictive conditions on $f(z,\cdot)$.

Key words and phrases. Indefinite potential, regularity theory, nodal solutions, Carathéodory reaction, superlinear near zero, Robin boundary condition.
2010 AMS Subject Classification: 35J20, 35J60.