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Consensus of nonlinear multi-agent systems with directed switching graphs: A directed spanning tree based error system approach[☆]

Zhiyong Yu, Haijun Jiang *, Da Huang, Cheng Hu

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, People's Republic of China

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ABSTRACT

The leaderless consensus problem of multi-agent systems with nonlinear dynamics and directed switching communication graphs is considered in this paper. The assumption in previous work that each possible communication graph contains a directed spanning tree is relaxed in this paper. Based on the directed spanning tree, we propose an error system which well transforms the consensus problem into the stabilization problem. By using matrix analysis theory and stability analysis of the nonlinear systems, a new kind of multiple Lyapunov function which depends on the communication graphs, is designed for analyzing the consensus behavior of the system. It is theoretically shown that the consensus can be achieved if the coupling gain is carefully chosen and other threshold conditions based on the communication graphs are satisfied. Finally, an example is presented to illustrate the theoretical analysis.

1. Introduction

Distributed cooperative control of multi-agent systems (MASs) has been intensively studied in the past few years. As one of the most fundamental research issues in the MASs, the consensus problem has attracted substantial attention due to its extensive applications in spacecraft formation control, traffic control, cooperative control of mobile autonomous robots, sensor networks and other areas [1–7]. The main task of the consensus problem is to devise communication protocols based only on local interaction making the states of all agents reach an agreement on a common value of interest [2,8,9].

A pioneering work on the consensus is [8], in which the authors provided a theoretical explanation for the heading consensus behavior from the Vicsek's model [10] by using tools from algebraic graph theory. Based on the Lyapunov approach, a general framework of the first-order consensus problems with fixed and switching topologies was proposed in [11]. Furthermore, some more relaxed conditions for consensus with switching topologies were presented in [2,12]. Since then, various control protocols and control strategies have been developed for MASs. For instance, feedback control [2,11,13], pinning control [14–16], intermittent control [17–19], impulses control [20–22], sampled data control [23,24], event-triggered control [25–27], adaptive control [28–30], and so on. Existing consensus algorithms can be roughly categorized into leaderless consensus and leader–follower consensus. Based on the communication topology, the consensus protocols can be divided into fixed topology protocols and switching topologies protocols.

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* Corresponding author.
E-mail address: jianghai@xju.edu.cn (H. Jiang).

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In [31], an infinite matrix products method was proposed to investigate the average consensus of the MASs with switching topologies. By using properties of non-negative matrices, the consensus of discrete-time MASs with switching topologies and time-varying delays was considered in [32]. In [33], the consensus of MASs with delays and uncertain switching topologies was studied. The leader-following consensus of nonlinear agents in hybrid varying directed topology was considered in [34]. The distributed consensus problem for multi-agent systems with unknown time delays and directed intermittent communications was investigated [35]. In [36], the cluster consensus of high-order MASs with general dynamics and switching topologies was investigated. The average consensus problems of the discrete-time Markov switching linear MASs with fixed topology and time-delay were studied in [37]. The continuous and discrete time consensus of MASs with linear time-invariant agent dynamics over randomly switching topologies was considered, in which the network topologies were assumed to be balanced [38]. By introducing a state transformation, the consensus problem of second-order MASs under both fixed and switching directed topologies was considered in [39]. The couple-group consensus problem for second-order discrete-time MASs was investigated in [40]. In [41], the authors studied the consensus of MASs with general linear dynamics under fixed and switching topologies. The global H_∞ consensus of discrete-time MASs was studied by designing event-based control strategy [42]. In [43], the authors considered the leaderless consensus of multi-agent systems with Lipschitz nonlinear dynamics and switching topologies, where each possible topology contains a directed spanning tree. In [44], the average consensus of second-order integral MASs under switching topologies and communication noises was studied by using distributed sampled-data based protocol. Under the assumption that the network topology is kept weakly connected and balanced, the second-order consensus for MASs with switching topologies and communication delay was investigated in [45]. The second-order locally dynamical consensus of MASs with arbitrarily fast switching directed topologies was considered in [46].

In order to analyze the consensus behavior, in most instances, the consensus problem is converted into a stabilization problem of the error system. For the leader–follower consensus, the error system between the leader and all followers can be easily obtained. For the leaderless case, the final consensus states of all agents are unknown a prior in most cases. Hence, the analysis of leaderless consensus is more difficult compared with the leader–follower case. Although a lot of results have been obtained on the consensus of MASs with switching topologies, summing up the existing research results, we find that the models are always simple. For example, in [31,33,36–38,41,42], they assumed that agents' intrinsic dynamics can be modeled by linear-invariant systems, and in [32,40,44,45], they supposed that the agents do not have intrinsic dynamic behavior. We also find that the network topology is very special such as undirected, strongly connected, balanced [35,37,38,45], each possible topology contains a directed spanning tree [43] and so on. However, in reality applications, the evolution dynamics of most real agents are indeed nonlinear, and in order to reduce communication costs, we should find the weakest possible condition that need to be imposed on the communication graph. Therefore, in this paper, we consider the leaderless consensus for MASs with Lipschitz-type nonlinear dynamics and directed switching topologies. One challenge is how to transform the consensus problem into the stabilization problem of the error system. The other one is how to design protocol and find the weakest possible condition that need to be imposed on the communication graph.

Motivated by aforementioned works, this paper aims to solve these problems. Hence, we consider the leaderless consensus for MASs with Lipschitz-type nonlinear dynamics over the directed switching communication graphs. The main contributions include the following. (1) The assumption in the existing works that each possible communication graph contains a directed spanning tree is relaxed, and we only suppose that the communication graphs contain a directed spanning tree in some bounded nonoverlapping time intervals in this paper. (2) Each agent has nonlinear dynamics. In order to study the consensus behavior, some useful lemmas are given and proved. (3) An error system based on the directed spanning tree is proposed, which well transforms the consensus problem into the stabilization problem of the error system. (4) Using the matrix theory and stability analysis, a new kind of Lyapunov function which depends on the communication graphs, is constructed for the switching systems. It is shown that the leaderless consensus can be achieved if the control gain is carefully selected and other threshold conditions about the communication graphs are satisfied.

The rest of this paper is organized as follows. In Section 2, some preliminaries in algebraic graph theory, necessary definition and lemma are provided. In Section 3, a reordering scheme for all nodes and some useful lemmas are given. In addition, the consensus for MASs with nonlinear dynamics and switching communication graphs is investigated. In Section 4, an example is presented to show the effectiveness of the theoretical results. A short conclusion is given in Section 5.

Notations. In this paper, let $\mathbb{N}$ be the set of the positive natural numbers. Let $R^{N\times N}$ be the set of real matrices. R^n denotes the n-dimensional Euclidean space. Let I_N be the identity matrix with dimension N, and $\mathbf{1}_N$ ($\mathbf{0}_N$) be the N-dimensional column vector with each entry being 1(0). For a square matrix A, let A^T represent its transpose, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ be the minimum eigenvalue and maximum eigenvalue of A, respectively. For a real symmetric matrix B, $B > 0$ ($B < 0$) if B is positive (negative) definite. For a matrix C, Rank(C) denotes the rank of C. $\|\cdot\|$ and $\otimes$ represent the Euclidean norm and the Kronecker product, respectively. diag($\cdot$) represents the diagonal matrix.

2. Preliminaries

Suppose that the considered MAS is composed of N agents. If each agent is considered as a node, then the MAS can be regarded as a network where the network topology is described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, in which $\mathcal{V} = \{v_1, \ldots, v_N\}$, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and $\mathcal{A} = [a_{ij}]_{N\times N}$ are the node set, the edge set and the weighted adjacency matrix, respectively. $e_{ij} = (v_i, v_j)$ is the edge of $\mathcal{G}$, and we refer to v_i and v_j as the parent node and child node of the edge (v_i, v_j), respectively. The neighbors set