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Orbit estimation using a horizon detector in the presence of uncertain celestial body rotation and geometry

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ABSTRACT

This paper presents an orbit estimation using non-simultaneous horizon detector measurements in the presence of uncertainties in the celestial body rotational velocity and its geometrical characteristics. The celestial body is modeled as a tri-axial ellipsoid with a three-dimensional force field. The non-simultaneous modelling provides the possibility to consider the time gap between horizon measurements. An unscented Kalman filter is used to estimate the spacecraft state variables and the geometric characteristics as well as the rotational velocity vector of the celestial body. A Monte-Carlo simulation is implemented to verify the results. Simulations showed that using non-simultaneous horizon vector measurements, the spacecraft state errors converge to zero even in the presence of an uncertain geometry and rotational velocity of the celestial body.

1. Introduction

Autonomous orbit estimation is a key element of modern space missions. For planet Earth, the use of the Global Positioning System (GPS) for the orbital navigation at low altitudes [1–3] is conventional. For high altitude missions the use of similar constellation-based navigation methods is proposed and tested as well [4,5]. However, the use of GPS does not make the satellite completely autonomous, since it is related to the constellation of the GPS satellites and the constellation is mostly navigated from ground stations [6]. On the other hand, relative states of two (or more) satellites can be utilized for an orbit estimation, independent of GPS satellites and/or ground stations [7–11]. Additionally, natural properties of a planet, like its magnetic field [12,13], atmosphere [14,15], or moons [16], can help to build an autonomous orbit estimation procedure. Spacecraft navigation and determination of Celestial Body (CB) characteristics can be autonomously accomplished using the planet's geometric characteristics [17] or gravity field estimation [18].

Horizon detectors are known for their ability of determining the nadir vector. For nadir-pointing satellites, the nadir vector is frequently utilized as a measurement to estimate the attitude [19]. Furthermore, the nadir vector can be used to estimate the satellite orbit as well. For Earth orbiting satellites, horizon detectors have been used for orbit determination purpose assuming spherical [20–24], and non-spherical Earth models [25]. Moreover, horizon sensors can be employed for finding the solar direction as discussed in Ref. [26].

In this paper, an autonomous orbit estimation using discrete non-simultaneous horizon detector measurements is addressed. Additionally, it is shown that these measurements can be utilized in the estimation of CB parameters; such as the semi-principal axes lengths and the angular velocity. The CB is modeled as a tri-axial ellipsoid, which is acceptable for most CBs in the solar system. The Unscented Kalman Filter (UKF) [27–29] is utilized for the estimation of the state and parameters in the presence of sensors noise and disturbances. The performance of this state and parameter estimation has been verified by the Monte-Carlo simulation. Thus, the main contributions of the paper are: (1) Unlike the previous investigations the time delays between horizon vector measurements are included, so the measurements are non-simultaneous; (2) the CB is modeled as a tri-axial ellipsoid with uncertain geometric characteristics that are augmented to the process model and estimated using parameter estimation; (3) similarly, the rotation of the CB about its primary axes is considered as an unknown and estimated in the filtering procedure; (4) MacCullagh's formula [30] is assumed as the governing gravitational dynamic model in the three-dimensional force field; (5) for such a problem a measurement model is proposed as an algorithm and UKF is utilized to overcome the non-linearities.

The rest of the paper is organized as follows: First, the process model is formulated using relative dynamics and MacCullagh's formula as the gravitational model. Next, the measurement model is derived and proposed as a unified algorithm. Section 4 reviews the UKF algorithm and Section 5 includes the simulation results. Finally, concluding remarks are presented.

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2. Process model

It is assumed that the geometry and the rotational velocity of the CB are not exactly known. Thus, by assuming the CB is a tri-axial ellipsoid, the semi-principal axes lengths (a, b, c), and its rotational velocity vector $(\boldsymbol{\omega})$ are included in the state vector of the system for the estimation purpose. In this manner, the process model can be summarized as the following equation:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \boldsymbol{w} \tag{1}$$

where $\mathbf{x} \triangleq [\mathbf{r}^T \;\; \dot{\mathbf{r}}^T \;\; \boldsymbol{\omega}^T \;\; a \;\; b \;\; c]^T$ is the state vector including $\mathbf{r}$ and $\dot{\mathbf{r}}$ as the position and velocity vectors of the spacecraft from the CB center of mass [31]. The state vector is augmented by the CB angular velocity and its semi-principal axes lengths to be estimated in the filtering procedure. A Gaussian, zero-mean white process noise, $\boldsymbol{w} \sim \mathcal{N}(0, \;\mathcal{Q})$, with a time-invariant covariance, $\mathcal{Q}$, is linearly added to the system of equations. The vector function, $\mathbf{f}(\mathbf{x})$, as the system differential equation is defined as

$$\mathbf{f}(\mathbf{x}) = \begin{Bmatrix} \dot{\mathbf{r}} \\ -\frac{\mu}{\|\mathbf{r}\|^3} + \mathbf{a}_{\text{eul.}} + \mathbf{a}_{\text{cor.}} + \mathbf{a}_{\text{cen.}} + \mathbf{a}_{\text{dis.}} \\ \mathscr{I}^{-1}(\boldsymbol{\omega} \times \mathscr{I}\boldsymbol{\omega}) \\ [0]_{3\times1} \end{Bmatrix} \tag{2}$$

in which the state vector $\mathbf{x}$ is defined in a coordinate system associated with frame A, attached to the CB, a Celestial Body-fixed Coordinate System (CBCS). The Euler acceleration resulting from angular acceleration, $\mathbf{a}_{\text{eul.}} = -\dot{\boldsymbol{\omega}} \times \mathbf{r}$, the Coriolis acceleration, $\mathbf{a}_{\text{cor.}} = -2\boldsymbol{\omega} \times \dot{\mathbf{r}}$, and the centrifugal acceleration, $\mathbf{a}_{\text{cen.}} = -\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$, are added to the two-body dynamics. The angular velocity $\boldsymbol{\omega} \equiv \boldsymbol{\omega}^{A/I}$ is defined as the rotation of the frame A with respect to the inertial frame, I. The disturbance acceleration, $\mathbf{a}_{\text{dis.}}$, is defined using MacCullagh's formula [30]:

$$\mathbf{a}_{\text{dis.}} = G\left(\frac{3}{2}\frac{\text{tr}(\mathscr{I})}{\|\mathbf{r}\|^5}[I]_{3\times3} + 3\frac{\mathscr{I}}{\|\mathbf{r}\|^5} - \frac{15}{2}\frac{\mathbf{r}^T\mathscr{I}\mathbf{r}}{\|\mathbf{r}\|^7}[I]_{3\times3}\right)\mathbf{r} \tag{3}$$

for a CB with moments of inertia matrix $\mathscr{I}$.

3. Measurement model

The horizon sensor is used for the purpose of this study. Thus, the measurement is based on the horizon unit vector defined in an inertial coordinate system, $\mathbf{u}$. It is assumed that the attitude of the satellite has been determined by alternative sensors such as star trackers and is perfectly known. Thus, the horizon unit vector can be found in the inertial frame. This horizon unit vector is modeled by a pair of spherical angles. Therefore, the measurement model can be written as

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\nu} \tag{4}$$

in which $\mathbf{z} \triangleq [\theta \;\; \phi]^T$ is the measurement output vector, and is defined to be the spherical angles of :

$$\mathbf{u} = \begin{Bmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ \sin\theta \end{Bmatrix} \tag{5}$$

as $\phi = \tan^{-1}(u_y/u_x)$, and $\theta = \sin^{-1}(u_z)$ (Fig. 1). The measurement Gaussian zero-mean white noise in Eqn. (4), $\boldsymbol{\nu} \sim \mathcal{N}(0, \;\mathcal{R})$, has a time-invariant covariance $\mathcal{R}$.

In order to define the measurement model, $\mathbf{h}(\mathbf{x})$, the formula of the horizon vector, $\mathbf{u}$, as a function of the position vector of the satellite, $\mathbf{r}$, should be found. If the unit vector, $\mathbf{u}$, is measured from the satellite at the point, $\mathbf{r}$, toward the ellipsoid horizon, the satellite position should be located on a quadratic surface of the following form:

$$\mathbf{r}^T Q\mathbf{r} + G = 0 \tag{6}$$

where,

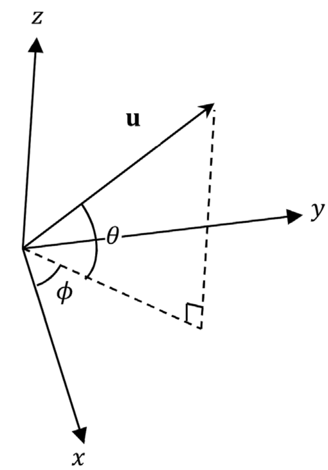

Fig. 1. Defining vector $\mathbf{u}$ in terms of spherical angles.

$$Q = L\mathbf{u}\mathbf{u}^T L - (\mathbf{u}^T L\mathbf{u})L$$

$$G = \mathbf{u}^T L\mathbf{u}$$

in which $L = \text{Diag}\{[1/a^2 \;\; 1/b^2 \;\; 1/c^2]^T\}$. Parameters a, b, and c are the lengths of the ellipsoid semi-principal axes. The derivation of Eqn. (6) is provided in Appendix I. However, it can be intuitively shown that the locus of the possible position vectors is a cylinder (Fig. 2).

Consider a horizon vector $\mathbf{u}^{RSW}$ defined in the RSW coordinate system. The RSW coordinate is defined such that its x axis is in the direction of the position vector $\mathbf{r}$, the z axis towards the orbital angular momentum vector of the satellite, and the y axis completes the right-handed coordinate system. The direction of the horizon vector is measured by the horizon sensor and the selected vector is not necessarily a unit vector. Introducing $\mathbf{u}^{RSW}$ by spherical angles yields:

$$\mathbf{u}^{RSW} \triangleq \left\|\mathbf{u}^{RSW}\right\| \begin{Bmatrix} \sin\theta^{RSW} \\ \cos\theta^{RSW}\sin\phi^{RSW} \\ \cos\theta^{RSW}\cos\phi^{RSW} \end{Bmatrix} \tag{7}$$

where θ^{RSW} and ϕ^{RSW} are defined with respect to the axes of the RSW coordinate system. The angle ϕ^{RSW} is assumed to be predefined for the satellite. In Eqn. (7), since the value of $\|\mathbf{u}^{RSW}\|$ is not assigned, it is assumed to be $\|\mathbf{u}^{RSW}\| = \sec\theta^{RSW}$ and then

$$u_R = \tan\theta^{RSW}, \quad u_S = \sin\phi^{RSW}, \quad u_W = \cos\phi^{RSW} \tag{8}$$

in which $\mathbf{u}^{RSW} \triangleq [u_R \;\; u_S \;\; u_W]^T$. The vector $\mathbf{u}^{CBCS} = [u_x^{CBCS} \;\; u_y^{CBCS} \;\; u_z^{CBCS}]$ defined in CBCS can be related to $\mathbf{u}^{RSW}$ as follows:

$$\mathbf{u}^{CBCS} = C_{RSW}^{CBCS}\mathbf{u}^{RSW} \tag{9}$$

in which C_{RSW}^{CBCS} is the rotation matrix from RSW to CBCS that is obtained from the estimated position and velocity vectors, and can be shown in the following form:

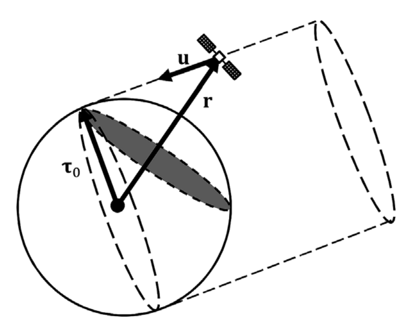

Fig. 2. A horizon unit vector measurement, $\mathbf{u}$, restricts the satellite position on an elliptic cylinder.