



# Arbitrary-step randomly delayed robust filter with application to boost phase tracking



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## ABSTRACT

The conventional filters such as extended Kalman filter, unscented Kalman filter and cubature Kalman filter assume that the measurement is available in real-time and the measurement noise is Gaussian white noise. But in practice, both two assumptions are invalid. To solve this problem, a novel algorithm is proposed by taking the following four steps. At first, the measurement model is modified by the Bernoulli random variables to describe the random delay. Then, the expression of predicted measurement and covariance are reformulated, which could get rid of the restriction that the maximum number of delay must be one or two and the assumption that probabilities of Bernoulli random variables taking the value one are equal. Next, the arbitrary-step randomly delayed high-degree cubature Kalman filter is derived based on the 5<sup>th</sup>-degree spherical-radial rule and the reformulated expressions. Finally, the arbitrary-step randomly delayed high-degree cubature Kalman filter is modified to the arbitrary-step randomly delayed high-degree cubature Huber-based filter based on the Huber technique, which is essentially an M-estimator. Therefore, the proposed filter is not only robust to the randomly delayed measurements, but robust to the glint noise. The application to the boost phase tracking example demonstrate the superiority of the proposed algorithms.

## 1. Introduction

Boost phase tracking, as an essential element of missile defense systems, has received a great deal attention in recent decades [1–3]. There are many reasons responsible for this instance, and the followings are the typical ones. In the first place, a lot of plume, which generated by the rocket engines, makes the detection of ballistic target more easily by the IR sensors [4,5]. In the second place, the trajectory prediction in coast flight phase mainly depends on the results of boost phase tracking [2]. However, boost phase tracking is still a challenging to the practitioner due to (1) motion modeling in boost phase, (2) glint noise in sensor measurements and (3) random delay of sensor measurements. To overcome these difficulties, some boost phase motion models and nonlinear filtering algorithms have been researched.

In general, the motion models used in boost phase tracking could be classified into two categories: profile-free and profile-based [6]. As for profile-based models, the model parameters such as orientation and magnitude of the thrust are assumed to be known, which are difficult to get in tracking non-cooperative target. Moreover, the tracking accuracy will decrease dramatically if the model approximation is poor. In contrast to the profile-based model, the profile-free models don't require the

additional information, such as the thrust profile and trajectory profile. Among the profile-free models, the gravity turn (GT) model is the most widely used one. This model is based on the hypothesis that the thrust is parallel to the velocity, and the tracking accuracy is fairly high if the hypothesis holds true in most of the time. Therefore, the GT model is utilized in this paper to model the motion of the ballistic target in boost phase.

As for nonlinear filtering algorithms, the most widely used one is the extended Kalman filtering (EKF). However, the EKF will perform worse or even divergence when the nonlinearity is severe since EKF is derived based on the first order linearization [7,8]. To improve the performance of filter in coping will nonlinear systems, the unscented transformation-based unscented Kalman filtering (UKF) [9,10], quadrature rule-based Gauss-Hermite filtering (GHF) [11], 3rd spherical-radial cubature (SRC) rule-based cubature Kalman filtering (CKF) [12,13] and polynomial interpolation-based divided difference filtering (DDF) [14] have been proposed in the past years. Recently, the arbitrary degree spherical-radial rule-based High-degree cubature Kalman filtering (HCKF), which is superior to UKF and CKF, has been proposed and applied to the tracking of ballistic target [15,16].

However, all the above filters are developed with two assumptions: 1.

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Abbreviations			
ARD	arbitrary randomly delayed	$GM$	gravitational parameter
CKF	unscented Kalman filtering	$J$	objective function
CQKF	cubature quadrature Kalman filtering	$p^{(i,j)}$	probability of $i$ time step delayed
DDF	divided difference filtering	$\bar{p}$	probability of no measurement received
EKF	extended Kalman filtering	$P$	covariance matrix
GHF	Gauss-Hermite filtering	$r$	position of the IR sensor
GT	gravity turn	$x$	state
HCHF	high-degree cubature Huber-based filter	$X$	augmented state
HCKF	high-degree cubature Kalman filtering	$y$	measurement without delay
IR	Infrared Radiation	$z$	actually received measurement
SRC	spherical-radical cubature	$Z$	measurements set
UKF	unscented Kalman filtering	$\gamma$	Bernoulli random variable
<i>Symbols and units</i>		$\mu$	turning parameter
$a$	magnitude of the net acceleration	$\xi$	cubature point
$b$	the ratio of reduced mass to total mass	$\rho$	real-valued function
		$\phi$	weights function

The measurement is available in real-time; 2. The measurement noise is Gaussian white noise. But in practice, both two assumptions are invalid since the measurements are always corrupted by the glint noise [17] and delayed by the congested network [18]. In this situation, the performance decreases seriously. To overcome the measurement delay problem, many researches have been carried out, which could be classified into two categories. The first category contains filters which are derived based on the backward prediction and forward prediction, such as Out-of-order sigma-point Kalman filtering [19], out-of-sequence high-degree cubature Huber-based filtering [20] and out-of-sequence Fokker-Planck filtering [21]. These filters perform well in coping with delayed measurements, but the latency time is necessary. To get rid of the dependence on latency time, some filters that belong to the second category were studied in recent years. Hermoso-Carazo proposed extended and unscented filtering algorithms for one [22] or two [23] randomly delayed measurements, where the random delay is modeled by the Bernoulli random variables. Later, the work is generalized to the Gaussian filter and correlated noise in Refs. [24,25]. However, all these filters are restricted to one or two step delays. Motivated by this problem, an arbitrary randomly delayed cubature quadrature Kalman filtering (ARD-CQKF) is proposed in Ref. [26] with an assumption that the probabilities of Bernoulli random variables taking the value one are equal, which is almost impossible in practice. Therefore, the paper extends the work and derives an arbitrary-step randomly delayed High-degree cubature Kalman filtering (ARD-HCKF) without this assumption. In addition, the non-Gaussianity of the measurement noise is ignored in the above studies, which will also decrease the accuracy of the filter. To address the estimation in such situation, a novel arbitrary-step randomly delayed High-degree cubature Huber-based filtering (ARD-HCHF) is proposed by combining the ARD-HCKF with a Huber technique, which is a combined minimum  $L_1$  and  $L_2$  norm estimator. Hence the ARD-HCHF is supposed to exhibit stronger robustness to

randomly delayed measurements and non-Gaussian noise.

Section 2 derives the ARD-HCKF with different probabilities of Bernoulli random variables taking the value one. In Section 3, derives the ARD-HCHF based on the combination of Huber technique and ARD-HCKF. In Section 4, the GT model and measurement model are introduced. The simulation is conducted in Section 5, and the conclusions are given in Section 6.

## 2. Arbitrary-step randomly delayed high-degree cubature Kalman filtering (ARD-HCKF)

### 2.1. Preliminaries

Consider a discrete-time stochastic nonlinear system described by

$$x_k = f_{k-1}(x_{k-1}) + \omega_{k-1} \tag{1}$$

$$y_k = h_k(x_k) + v_k \tag{2}$$

where  $x_k \in R^n$  is the  $n$ -dimensional state vector at time  $k$ ,  $y_k \in R^m$  is the  $m$ -dimensional measurement vector without delay,  $w_k \in R^n$  and  $v_k \in R^m$  are uncorrelated zero-mean Gaussian white noises satisfying  $E[\omega_k \omega_l^T] \in Q_k \delta_{kl}$  and  $E[v_k v_l^T] \in R_k \delta_{kl}$ , respectively, where  $\delta$  is the Kronecker delta function,  $f$  and  $h$  denote the known nonlinear state function and measurement function, respectively.

In practice, the measurements are always delayed by the congested network. If the maximum number of delay is  $N - 1$  steps (where  $N$  could be chosen by the practitioner), the measurements received at the  $k$ -th time may be  $y_{k-i}$  ( $0 \leq i \leq N - 1$ ) or nothing. If nothing is received, the measurements received at the  $(k - 1)$ -th time  $z_{k-1}$  will be utilized in the estimator. Therefore, the measurement equation could be modified as:

$$\begin{aligned}
 z_k &= (1 - \gamma_1)y_k + \gamma_1(1 - \gamma_2)y_{k-1} + \gamma_1\gamma_2(1 - \gamma_3)y_{k-2} + \dots + \left(\prod_{i=1}^{N-1} \gamma_i\right)(1 - \gamma_N)y_{k-N+1} \\
 &+ \left[1 - (1 - \gamma_1) - \gamma_1(1 - \gamma_2) - \gamma_1\gamma_2(1 - \gamma_3) - \dots - \left(\prod_{i=1}^{N-1} \gamma_i\right)(1 - \gamma_N)\right]z_{k-1} \\
 &= \gamma^{(0,j)}y_k + \gamma^{(1,j)}y_{k-1} + \gamma^{(2,j)}y_{k-2} + \dots + \gamma^{(N-1,j)}y_{k-N+1} + \left(1 - \sum_{i=0}^{N-1} \gamma^{(i,j)}\right)z_{k-1} \\
 &= \sum_{i=0}^{N-1} \gamma^{(i,j)}y_{k-i} + \left(1 - \sum_{i=0}^{N-1} \gamma^{(i,j)}\right)z_{k-1}
 \end{aligned} \tag{3}$$

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