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Geomagnetic field models for satellite angular motion studies

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ABSTRACT

Four geomagnetic field models are discussed: IGRF, inclined, direct and simplified dipoles. Geomagnetic induction vector expressions are provided in different reference frames. Induction vector behavior is compared for different models. Models applicability for the analysis of satellite motion is studied from theoretical and engineering perspectives. Relevant satellite dynamics analysis cases using analytical and numerical techniques are provided. These cases demonstrate the benefit of a certain model for a specific dynamics study. Recommendations for models usage are summarized in the end.

1. Introduction

Magnetic attitude control systems, both active and passive, are widely used for CubeSats and other small satellites. The first satellite with passive magnetic control was Transit 1B [1] launched April 13, 1960. The first satellite with active magnetic control was Tiros II [2] launched November 23, 1960. Magnetic field was first used for attitude determination aboard the third Soviet satellite [3] launched May 15, 1958. Novel small satellites and especially CubeSats actively utilize the same principles. Here we outline some basic and modern works concerning magnetic attitude control.

Angular velocity damping is the main task for most magnetic control systems. The first concept was to use hysteresis rods [1,4,5]. This simple approach is still popular [6–8]. Eddy current damper [9] with/without viscous fluid has no use now.

Active magnetic control systems are preferred aboard modern satellites. Magnetorquers have low cost, mass, power consumption and can be easily used even on CubeSats. "-Bdot" is the most common magnetic control algorithm. Published in Ref. [10] and first mentioned in Ref. [11], this algorithm was proposed by GSFC engineer Seymor Kant. Its investigation and in-flight performance still attract interest [12–17].

Magnetic control provides specific attitude motion utilizing the spin stabilization approach or auxiliary actuators. These are necessary to overcome the underactuation issue: there is no control authority along the geomagnetic induction vector. Common attitude control schemes of spin stabilized satellites were proposed in Refs. [18,19]. Remarkable examples of analysis or implementation can be found in Refs. [2,10, 20–25]. Spin stabilization allows promising optimal reorientation problem statement [26–28].

Auxiliary actuators, mainly the gravity-gradient boom or the flywheel with constant rotation rate [29–32] provide passive control authority necessary for stabilization in orbital reference frame. Fully magnetic control system may be used to provide any necessary attitude [33–38]. This relatively new and largely uncharted area is of special interest for small satellites.

Geomagnetic field model is necessary for attitude control and/or system design, on-board control computation and attitude determination process (if magnetometer is used). Even passive magnetic system may require a model since attitude determination is still necessary for payload data interpretation. This paper focuses on models relevant for these applications. Four important models are introduced with the exact expressions in different reference frames. Examples are discussed for analytical and numerical analysis. Final recommendations are provided regarding different models implementation for specific purposes.

2. Geomagnetic field models

2.1. Reference frames

$O_aY_1Y_2Y_3$ is an inertial frame (though it is not, this assumption is commonly accepted). O_a is Earth's center, O_aY_3 axis is directed along Earth's daily rotation axis, O_aY_1 lies in the equatorial plane and is directed to the ascending node of the circular orbit, O_aY_2 is directed so that the reference frame is right handed (the same holds for the following frames).

$O_aZ_1Z_2Z_3$ is an inertial reference frame. It is got from $O_aY_1Y_2Y_3$ by

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turning by the angle Θ about O_aY_1 axis. This angle is defined with the simplified dipole model.

$O_aS_1S_2S_3$ inertial frame is bound to the satellite's orbit. Axis O_aS_3 is normal to the orbital plane (directed along the orbital angular velocity). O_aS_1 axis is directed to the ascending node. Transition between frames $O_aY_1Y_2Y_3$ and $O_aS_1S_2S_3$ is represented with the rotation by the angle i (orbit inclination) about O_aY_1 axis. Transition between frames $O_aS_1S_2S_3$ and $O_aZ_1Z_2Z_3$ requires the rotation by the angle $\Theta - i$ around O_aS_1. Inertial reference frames are depicted in Fig. 1.

$OX_1X_2X_3$ is the orbital reference frame centered in the satellite's center of mass. Axis OX_1 is directed along the radius vector $\overrightarrow{O_aO}$. OX_2 axis lies in the orbital plane. It is perpendicular to the radius-vector of the satellite and points toward the orbital motion direction. OX_3 is normal to the orbital plane.

$Ox_1x_2x_3$ is the bound reference frame, its axes coincide with the principal axes of inertia of the satellite.

$OL_1L_2L_3$ frame is bound to the satellite's angular momentum vector. OL_3 axis is directed along this vector. OL_2 axis lies in the plane of two first axes of inertial frame. Any inertial frame may be used depending on convenience and mission requirements of the satellite motion representation (Fig. 3).

Vectors are supplemented with lower indices of corresponding reference frames where necessary. For example, the geomagnetic induction vector may be written as $\mathbf{B}_Y$ in $O_aY_1Y_2Y_3$ frame. Direction cosines matrices are

$$\mathbf{A}_{YZ} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta & \sin\Theta \\ 0 & -\sin\Theta & \cos\Theta \end{pmatrix}, \quad \mathbf{A}_{YS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{pmatrix},$$

$$\mathbf{A}_{SX} = \begin{pmatrix} \cos u & \sin u & 0 \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{A}_{YX} = \mathbf{A}_{SX}\mathbf{A}_{YS}$$
$$= \begin{pmatrix} \cos u & \sin u\cos i & \sin u\sin i \\ -\sin u & \cos u\cos i & \cos u\sin i \\ 0 & -\sin i & \cos i \end{pmatrix}$$

where u is the argument of latitude. Transition rule is $\mathbf{x}_j = \mathbf{A}_{ij}\mathbf{x}_i$.

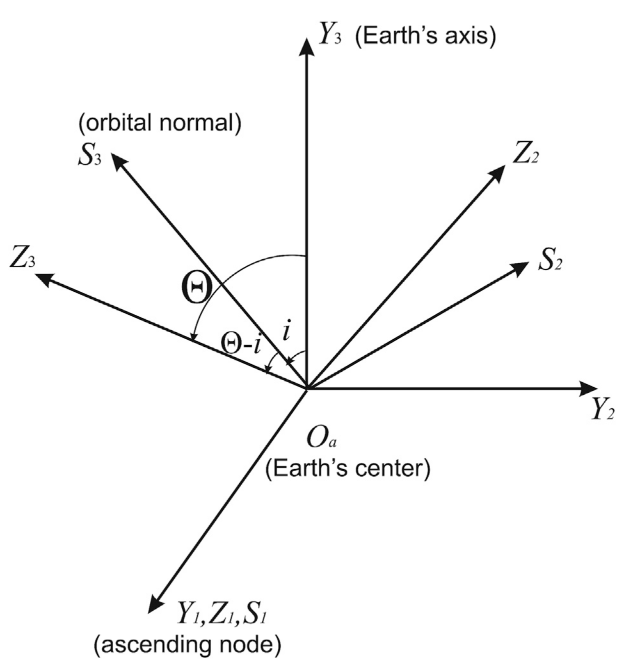

Fig. 1. Inertial reference frames.

2.2. International geomagnetic reference field/World magnetic model

IGRF (International geomagnetic reference field) and WMM (World magnetic model) are the most accurate models. Field decomposition is used in both models. It was proposed by C.F. Gauss in 1838. The decomposition is

$$V = -R\sum_{n=1}^{N}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\left(g_n^m(t)\cos m\lambda_0 + h_n^m(t)\sin m\lambda_0\right)P_n^m(\cos\vartheta_0), \quad \mathbf{B} = \nabla V$$

where λ_0 is the longitude of the point where the induction vector is calculated, $\vartheta_0 = 90^\circ - \theta_0$, θ_0 is the latitude of the point, r is the distance to the point from Earth's center, R is the average Earth radius. g_n^m and h_n^m are the Schmitt coefficients given in a table [39], P_n^m is a quasinormalised Legendre polynomial. Coefficients and decomposition terms limit N are derived empirically for both models (coefficients and the resulting geomagnetic induction are given in nT). They are valid for five years. Coefficients are updated by the International Union of Geodesy and Geophysics for IGRF model and by the USA National Oceanic and Atmospheric Administration for WMM. Both models are designed for altitudes of no more than 600 km (relative to World Geodetic System reference ellipsoid WGS84) though they may be actually used for higher orbits. These accurate models are often used onboard and in numerical simulation. IGRF is more widespread for satellite motion applications.

2.3. Inclined dipole model

Inclined (or tilted) dipole represents the major part of Gauss model. First order terms ($N = 1$) are taken into account [40,41]. These terms describe the dipole tilted by a small angle with respect to the opposite direction of Earth's rotation axis. Exact inclination angle varies as Earth's magnetic poles drift. This value is approximately 9.35° now. Dipole approximation is enough for most cases of artificial satellite motion. The inclined dipole model takes into account two main sources of geomagnetic field variation: the satellite motion along the orbit and Earth's rotation. Irregular effects (for example regions above the highly magnetized areas) are not accounted for. The dipole geomagnetic induction vector in any reference frame is

$$\mathbf{B} = -\frac{\mu_e}{r^5}\left(\mathbf{k}r^2 - 3(\mathbf{kr})\mathbf{r}\right)$$

where $\mathbf{k}$ is a unit dipole direction, $\mathbf{r}$ is satellite's radius vector, $\mu_e = \mu_0\mu_m/4\pi$ is derived from three first coefficients in the Gauss decomposition, μ_m is the dipole value (currently μ_e =7.7245 T km^3), $\mu_0 = 4\pi\cdot 10^{-7}$ kg•m•A^{-2}•s^{-2} is the magnetic constant. The inclined dipole allows rather compact representation in $O_aY_1Y_2Y_3$ frame,

$$\mathbf{B}_Y = -\frac{\mu_e}{r^3}\begin{pmatrix} \sin\lambda_2\sin\delta_1 - 3\xi\cos u \\ -\cos\lambda_2\sin\delta_1 - 3\xi\cos i\sin u \\ \cos\delta_1 - 3\xi\sin i\sin u \end{pmatrix} \tag{1}$$

Angles λ_2, δ_1 provide the dipole attitude with respect to $O_aY_1Y_2Y_3$ frame. Angle $\lambda_2 = \omega_E t + \lambda_{20}$ where ω_E is Earth's rotation rate represents the dipole rotation with respect to $O_aY_1Y_2Y_3$, λ_{20} depends on the initial time moment, $\xi = \cos u\sin\delta_1\sin\lambda_2 - \sin u\cos i\sin\delta_1\cos\lambda_2 + \sin u\cos\delta_1\sin i$, $\delta_1 \approx 170.65^\circ$. The inclined model provides quite bulky expressions in other reference frames. Transition matrices are often used instead of the resulting expressions.

2.4. Direct dipole model

Geomagnetic field model is further simplified by the direct dipole. The dipole unit vector is represented in $O_aY_1Y_2Y_3$ frame as $\mathbf{k} = (0, 0, -1)$. The induction vector (1) is