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Theory of the vibrational hydrodynamic top ☆

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ABSTRACT

Dynamics of a viscous fluid is investigated theoretically in an annulus with the free inner cylinder under conditions of rotation in an external inertial or gravitational field. The two-dimensional formulation is used, corresponding to two long coaxial cylinders. The inner cylinder is free and occupies a steady position on the rotation axis under the action of the centrifugal force due to the fact that it is lighter than surrounding fluid. The action of external force, oriented perpendicular to the rotation axis, induces inertial circular, of the tidal-like type, oscillations of the inner cylinder (core). As a result of the oscillations, the core is brought into rotation relative to the cavity (the outer cylinder) on the background of a steady streaming in the annulus. The mechanism of this differential rotation consists in the generation of an average mass force, of the azimuthal direction, in the oscillating viscous boundary layers on the walls of the core and the cavity.

1. Introduction

Rotating systems are widely spread in nature and technology. Here, important role belongs to the inertia forces: centrifugal and Coriolis forces. Due to their action the rotating fluid acquires non-trivial properties, not found in the absence of rotation. Speaking about momentum and energy transfer in a rotating fluid, it is worth to distinguish viscous interaction in boundary layers on the walls of a container or a body moving in fluid, and inertial waves [1].

Examples of rotating hydrodynamic systems are celestial bodies: stars and planets. There are planets with a solid inner core surrounded by a fluid, e.g. a liquid core in case of the Earth. In the field of an external massive satellite, inside the planet tidal oscillations may be excited, influencing significantly the dynamics of the core. A simple model of such situation, e.g. influence of a satellite on a planet's solid core and liquid core motion, is the rotation of a two-phase system – comprising a solid core and a liquid shell – in an external inertial field.

Vibrations are an actual topic both for fundamental science and technology [2]. Circular vibrations of the inner cylinder in an immobile outer cylinder generate fluid oscillations in the annulus. In the presence of the temperature difference between the surfaces of the cylinders, in the fluid a non-viscous average “vibrational” force is generated, directed radially [3]. This leads to the generation of a steady azimuthal flow. In [4] circular vibrations of quasi-concentric cylinders are reported to induce intensive vortical motion of a viscous fluid filling the annulus.

It is known that fluid oscillations near a solid surface or an interface lead to the generation of mean flows in Stokes layers, known as steady streaming [5,6] or acoustic streaming [7].

In the study of dynamics of a fluid layer with the free surface in a partially filled rotating cylinder, the radial gas column displacement under the gravity action is found, stationary in the laboratory frame [8]. Meanwhile, the surface retains the circular cross section, and its motion in the cavity frame represents circular oscillations. In consideration

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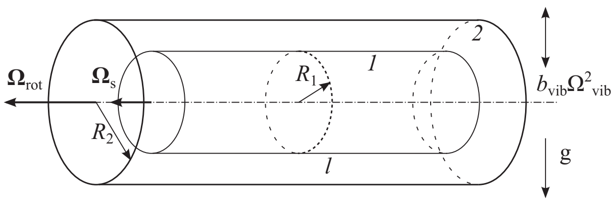

Fig. 1. The coaxial layer: 1 – the body (radius R_1), 2 – the cavity (radius R_2).

of this problem from the positions of vibrational hydromechanics [9] it is shown that such inertial oscillations of a fluid layer lead to the generation of a steady streaming in a Stokes boundary layer. Effect of vibrations, perpendicular to the rotation axis, on the described system leads to a resonant excitation of surface oscillations resulting in the steady streaming [10].

In [11] the rotating cylinder containing a "weightless" free cylinder and entirely fluid-filled is studied experimentally and theoretically. Differential rotation of the inner cylinder is reported and an analytical solution is made, which overestimates rotation velocity measured in the experiment.

The action of an external force, perpendicular to the rotation axis, on a free inner cylinder in a rotating outer cylinder with liquid excites circular oscillations of the first. Due to generation of a steady streaming in the viscous boundary layers, the inner cylinder spins [12]. We name a body rotating fast relative to a rotating fluid under the action of a vibrational force "vibrational hydrodynamic top".

In the presented work, the role of viscous exchange between the liquid and solid is considered in conditions of inertial tidal oscillations of the solid core, as well as the influence of oscillating viscous boundary layers on the flow in a coaxial layer. The problem is solved in a two-dimensional formulation, however it will be shown that a significant part of parameters does not depend on geometry and may be applied for description of dynamics of both cylindrical and spherical bodies.

The problem of fluid motion in a coaxial layer formed by two cylinders and subject to transverse vibrations was studied experimentally and theoretically in [12]. The boundary conditions used in the cited work are valid for the case of the relative radius $R = R_1/R_2$ close to unity, e.g. $R=0.9$, and underestimate the intensity of the core differential rotation for $R \approx 0.6$, used in experiments, by approximately 2. In the present work a more precise boundary condition is introduced for finding the velocity of fluid oscillations beyond the boundary layer. This allows us to extend the theory validity to an arbitrary value of R.

2. Problem formulation

The inner cylinder 1 (the body) is free and lighter than the liquid. The outer cylinder 2 is at constant rotation with the angular velocity Ω_{rot}. Under rotation the body is positioned on the rotation axis under the action of centrifugal force (Fig. 1). The layer is assumed infinitely long, so that the derivative with respect to the axial coordinate is zero, and the effect of the ends is neglected. To achieve this in the experiment, in most cases the aspect ratio of the layer $l' \equiv l/(R_2 - R_1)$ is of the order of 10. Here, l is the length of the body. Thus, the solution reduces to finding the velocity field in a two-dimensional layer of isothermal and homogeneous in density fluid (Fig. 2). The described system is subject to the action of an external inertial field, static or vibrating in the laboratory frame with the frequency Ω_{vib}. The vector of the external force is perpendicular to the rotation axis. In the cavity frame the external force rotates with the frequency $\Omega_{osc} = \Omega_{vib} - \Omega_{rot}$ and excites body circular oscillations. In the case of a static external force, $\Omega_{vib} = 0$ and $\Omega_{osc} = -\Omega_{rot}$.

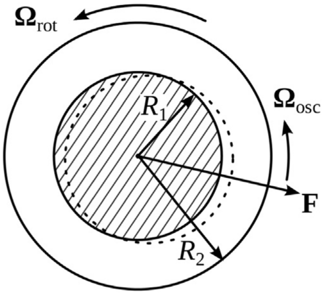

Fig. 2. The annulus in the cross section.

The body oscillation amplitude is supposed to be small compared to the annulus gap and to the body size:

$$b_s \ll R_2 - R_1, \quad b_s \ll R_1. \tag{1}$$

The problem is solved in the approximation of high oscillation frequency:

$$R_2 - R_1 \gg \sqrt{\frac{2\nu}{\Omega_{osc}}}. \tag{2}$$

This means that the viscous forces act only near the solid walls of the body and the cavity. Here, Ω_{osc} is the frequency, in the cavity frame, of body oscillations induced by an external oscillating force **F**.

The fluid is split into two domains: viscous, inside the Stokes boundary layers localized on the solid walls, and non-viscous, in the annulus beyond the boundary layers.

The fluid dynamics in rotating cavity is described by Navier–Stokes and continuity equations:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{\Omega}_{rot} \times \mathbf{r} \times \mathbf{\Omega}_{rot} + 2\mathbf{v} \times \mathbf{\Omega}_{rot},$$
$$\mathrm{div}\, \mathbf{v} = 0. \tag{3}$$

2.1. Experimental technique and results

The body 1 and the cuvette 2 (Fig. 1) are made of organic-glass tubes sealed from the ends. The volume between the