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Stress distribution analysis and optimization for composite laminate containing hole of different shapes

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A B S T R A C T

This paper proposes a flexible analytical method to obtain the circumferential stress of hole's edge in composite laminate, which can provide convenience for the optimization of hole's shape meanwhile. Firstly, different shapes of hole are expressed in one map function based on the conformal transformation. Secondly, stress function method is applied to the anisotropic elastic theory, which derives the stress expressions to calculate arbitrary point's stress in equivalent anisotropic plate. Thirdly, the circumferential stress of hole's edge in composite laminate can be calculated by combining the map function with the stress expressions. This method is verified by finite element simulation for the hole of circle, triangle and square, which draw a conclusion that circle hole is not always the best choice for aeronautical structure design. Meanwhile, an optimal shape of hole is obtained by the mentioned method with help of gradient descent algorithm and mesh deformation technique. The optimization result is validated by simulation and it meets the conclusion identically, which also means the method's accuracy is reliable.

1. Introduction

In aeronautical structure designs, the analysis of stress distribution is a common but essential section, as well as the optimization. The holes, which ensure pipelines or wires can pass through the cabin, are inevitable in design progress and bring problem of stress concentration into the analysis [1–7] and optimization. In the past, aircrafts were mainly made of metals. It was relatively easy to analyze and optimize the stress distribution in aeronautical structure containing hole, for the metals were isotropic and the hole was regular. However, with obvious advantages and superior performances shown, composite laminates have been used widely and frequently in aircraft manufactures [8,9]. Not only will fibers' breakage due to the hole cause stress concentration much complication, but the materials' anisotropy demands new method to analyze and optimize, for which traditional theories based on metals' experiments are not suitable.

For the analysis of stress distribution around hole in structure, extensive work has been done to explore the relationship between stress concentration and hole of different shapes. Savin [10] investigated an infinite anisotropic plate containing an oval hole. The infinite area out of hole was mapped into a unit circle, then analytical solution of stress could be obtained by integrating Schwarz equation in Cauchy method. Aiming to improve the efficiency of calculation, Lekhnitskii [11] applied series expansion method to the analytical solution and got an approximate solution. Theocaris [12] and Ukadgaonker [13] investigated the stress concentration of isotropic plate containing hole of equilateral triangle. The former combined Schwarz–Christoffel transformation with Muskhelishvili's complex-function theory and mapped the equilateral triangle with rounded-off corners into the unit circle. The later validated Savin's equation under different load. Daoust and Hoa [14] extended the work to anisotropic plate containing hole of arbitrary triangle while Grüber [15] and Nageswara [16] studied laminates containing oral hole. Bižić [17] researched a homogeneous isotropic disc weakened with an eccentric circular hole by using complex variable method. Kiani [18] dealt with annular plate of heated functionally graded material and paid much attention to the buckling behavior under thermal load. The equilibrium equations of annular-shaped plate were obtained based on the classical plate theory and each thermo-mechanical property of the plate was assumed to be graded across the thickness direction of plate based on the power law form, while Poisson's ratio was kept constant. In the previous studies, researchers mainly focused on calculating stress distribution under unique condition that anisotropic plate contained hole of special shapes, and there was not one systematic analysis which could fit the work conditions flexibly. Thus, it's necessary to propose a new method which can change the hole's shape freely so that the optimization can be done easily.

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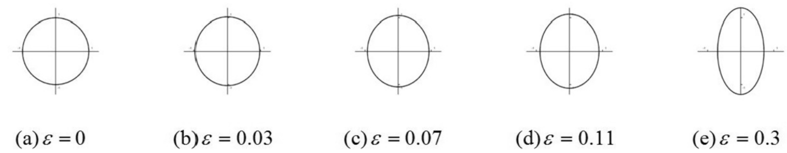

Fig. 1. Circle hole ($n = 1$) and its variation caused by ε.

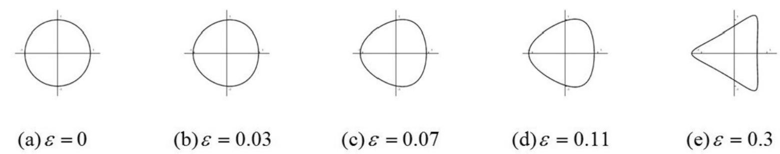

Fig. 2. Triangle hole ($n = 2$) and its variation caused by ε.

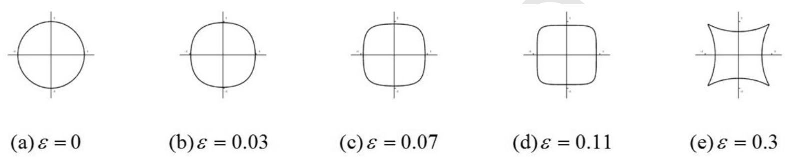

Fig. 3. Square hole ($n = 3$) and its variation caused by ε.

In this paper, the hole's shape is parameterized and expressed in one map function, firstly. Secondly, an analytical method of calculating stress distribution is obtained by substituting the map function into anisotropic elastic theory. Thirdly, finite element models are established in order to verify the method's accuracy. According to the conclusion, the hole's shape is optimized based on gradient descent algorithm and validated by simulation at last.

2. Theoretical analysis

2.1. Hole's map function

Based on conformal transformation in rectangular coordinate system, the hole's map function of different shapes can be expressed as

$$\begin{cases} x = \cos\theta - \varepsilon\cos(n\theta) \\ y = -\sin\theta - \varepsilon\sin(n\theta) \end{cases} \tag{1}$$

where x, y are coordinate parameters; ε, n are shape parameters while ε controls the curvature and n controls the hole's basic shape, which is shown in Fig. 1, Fig. 2 and Fig. 3; θ ranges from 0 to 2π.

2.2. Stress function method

The relationship between stain and stress in anisotropic material is

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{16} \\ S_{12} & S_{22} & S_{26} \\ S_{16} & S_{26} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \tag{2}$$

where ε_x, ε_y and γ_{xy} are the strain parameters; σ_x, σ_y and τ_{xy} are the stress parameters; S_{11}, S_{12}, S_{16}, S_{22}, S_{26} and S_{66} are the parameters of flexibility matrix.

The compatibility equation of deformation determined by strain and displacement is

$$\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} - \frac{\partial^2\gamma_{xy}}{\partial x\partial y} = 0 \tag{3}$$

The body force ignored, introduce the stress function $U(x, y)$ and it leads to

$$\begin{cases} \sigma_x = \dfrac{\partial^2 U(x,y)}{\partial y^2} \\ \sigma_y = \dfrac{\partial^2 U(x,y)}{\partial x^2} \\ \tau_{xy} = -\dfrac{\partial^2 U(x,y)}{\partial x\partial y} \end{cases} \tag{4}$$

Substitute Eq. (2) and Eq. (4) into Eq. (3), then it becomes

$$S_{22}\frac{\partial^4 U(x,y)}{\partial x^4} - 2S_{26}\frac{\partial^4 U(x,y)}{\partial x^3\partial y} + (2S_{12} + S_{66})\frac{\partial^4 U(x,y)}{\partial x^2\partial y^2} - 2S_{16}\frac{\partial^4 U(x,y)}{\partial x\partial y^3} + S_{11}\frac{\partial^4 U(x,y)}{\partial y^4} = 0 \tag{5}$$

Suppose μ_k ($k = 1, 2, 3, 4$) are the conjugate complex roots of Eq. (5)'s characteristic equation and

$$\begin{cases} \mu_1 = \alpha_1 + i\beta_1 \\ \mu_2 = \alpha_2 + i\beta_2 \\ \mu_3 = \alpha_1 - i\beta_1 \\ \mu_4 = \alpha_2 - i\beta_2 \end{cases} \tag{6}$$

where α_1, β_1, α_2, β_2 are real numbers and i is imaginary unit.

Suppose $z_i = x + \mu_i y$ ($i = 1, 2$) and introduce the complex variable function $F(z)$. Notice that the stress function must be a function of real numbers, so it can be expressed as

$$U = 2\,\mathrm{Re}\big[F_1(z_1) + F_2(z_2)\big] \tag{7}$$

Introduce intermediate functions $\varphi(z_1) = \frac{dF_1(z_1)}{dz_1}$ and $\psi(z_2) = \frac{dF_2(z_2)}{dz_2}$, then combine them with Eq. (7) and Eq. (4). Finally the stress function of anisotropic material can be obtained as

$$\begin{cases} \sigma_x = 2\,\mathrm{Re}\big[\mu_1^2\varphi'(z_1) + \mu_2^2\psi'(z_2)\big] \\ \sigma_y = 2\,\mathrm{Re}\big[\varphi'(z_1) + \psi'(z_2)\big] \\ \tau_{xy} = -2\,\mathrm{Re}\big[\mu_1\varphi'(z_1) + \mu_2\psi'(z_2)\big] \end{cases} \tag{8}$$

Now the stress can be calculated when proper function $\varphi(z_1)$ and $\Psi(z_2)$ are found.