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Interval analysis of the wing divergence

Yi Li *, Tianhong Wang

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, Shaanxi, China

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ABSTRACT

In this paper the Taylor expansion (TE) is combined with the optimization and anti-optimization problems (OAP) solutions of parameterized interval analysis (PIA) to study the effect of structural uncertainties on the divergence of wing. Through a two-dimensional wing example, the TE + PIA + OAP method developed by this paper is compared with the classic interval analysis (CIA), the results show that the TE + PIA + OAP method can reduce overestimation in the CIA and TE method, and the interval of divergence dynamic pressure predicted by the TE + PIA + OAP method is as narrow as the one from the PIA + OAP methods, but the computation cost of the TE + PIA + OAP method is much lower. In addition an actual engineering example of forward swept wing with uncertain structural parameters is studied by the TE + PIA + OAP method to illustrate the capability to solve the real divergence problem of wing with uncertain structural parameters.

1. Introduction

Aeroelastic stability (e.g. flutter and divergence) can be predicted by theoretical and simulation method with the structural and aerodynamic parameters, however most of structural parameters involved in these predictions are physically uncertain variables that arise from several sources, such as manufacturing tolerances, material differences, and wear. A study of the McDonnell Douglas F-4 Phantom II [1] quantified the weight and inertia variability for the frames, these changes in mass and inertias of control surfaces were up to 15%, thus aeroelastic analyses need to quantify and propagate these uncertain parameters in the aircraft structures, and this need had become increasingly pressing in recent years. Pettit [2] and Dai Yuting [3] gave a review of this subject. In these reviews, the general sources of uncertainty that complicate airframe design and testing were described, and the popular uncertainty quantification methods in aeroelasticity were given.

Uncertainty quantification technologies can be broadly categorized as probabilistic or non-probabilistic method [4]. In probabilistic approaches, the probability density functions of the uncertain parameters have to be properly defined, and can be quantified by several techniques such as the Monte Carlo Simulation [5] (MCS), perturbation method [6], Neumann expansion [7], Polynomial Chaos Expansion [8] and stochastic collocation [9], in recent five years Scarth, Cooper and Georgiou [4,10] used Polynomial Chaos Expansion method (PCE) to study the effect of uncertain material and lamination parameters on the aeroelastic behavior of composite wing. In non-probabilistic approaches the interval can be used to represent the uncertainty range, and the uncertain parameters can be quantified by the classic interval analysis [11]. However, the application of classic interval analysis to practical engineering problems is quite difficult because it may lead to large overestimation due to the so-called dependency phenomenon [12]. In order to limit the overestimation the generalized interval analysis [13] and the affine arithmetic [14] have been introduced by researchers. In the last decades, within the framework of static structural analysis, the improved interval analysis by the Extra Unitary Interval (EUI) [15], Interval Perturbation Method (IPM) [16], the Parameterized Interval Analysis (PIA) [17] and Taylor Expansion (TE) [18,19] have been developed. Specifically, Muscolino and Sofi [20] combined the Interval Rational Series Expansion (IRSE) with the Improved Interval Analysis by the Extra Unitary Interval (IIA-EUI) to quantify the uncertain parameters in static analysis of structural systems. In the same context, Elishakoff and Miglis [21] combined the parameterized interval analysis (PIA) with the optimization and anti-optimization problems (OAP) to solve the static structural problem with uncertain parameters. A new method is presented by Santoro, Muscolino and Elishakoff [22]. This method combined the PIA + OAP with the IRSE to overcome the limits of IIA-EUI + IRSE and PIA + OAP, and efficiently solved the static problem with uncertain parameters. Recently, Wang Xiaojun and Qiu Zhiping [23,24] studied the influences of uncertain structural parameters described by interval numbers on the flutter speed of wing. In addition to flutter, divergence is also threat to the integrity of the wing structure, especially for the forward

* Corresponding author.
E-mail address: liyi504@nwpu.edu.cn (Y. Li).

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swept wing, the divergence is more likely to occur, due to effect of the bend-twist coupling. And the uncertain structural parameters have an impact on the divergence dynamic pressure of the wing.

In this paper the uncertain parameters are described by interval numbers, effect of these uncertainties on the divergence is studied by combining the PIA + OAP with the TE. The divergence dynamic pressure of a two-dimensional airfoil with uncertain geometrical parameters and twist stiffness is calculated by the CIA, the PIA + OAP and the PIA + OAP + TE methods separately. The results show that the method developed here (the TE + PIA + OAP) can also obtain the narrow interval of divergence dynamic pressure as good as the PIA + OAP. Basing on the response at the central value of uncertainties and the sensitivities respect to these uncertain variables, the response of complicated system (such as wing structure) with uncertainties can be approximated by the first order Taylor series expansion, and then there is no need to rewrite the motion equation of aeroelastic system, consequently this method is more effective than the PIA + OAP method and can be easily applied for the real divergence problem of wing and aircraft. In order to demonstrate this ability, a classic forward swept wing with uncertain structural parameters is studied by the TE + PIA + OAP method, and the boundary of divergence dynamic pressure is obtained as well.

2. Divergence of the two dimensional wing

The phenomenon of divergence was explained by Fung through a two-dimensional wing [25], as shown in Fig. 1. The elastic restraint imposed on this wing can be regarded as a torsional spring through a point G which is fixed in space, the length of wing is unit span. The wing is first rotated through an angle α as a rigid body, then deflected elastically through an additional angle θ. It is desired to find the equilibrium position of the wing in a flow of speed U. The aerodynamic force on the wing can be represented by a lift force, acting through the aerodynamic center, and a moment about the same point. The distance from the aerodynamic center to the axis of the torsional spring can be written as ec, the chord length is c, e is a ratio expressing the eccentricity of the aerodynamic center (positive if the torsional spring lies behind the aerodynamic center). The twisting angle of a two-dimensional wing can be given by Eq. (1) [25].

$$\theta = \frac{qec^2 a\alpha}{K_\alpha - qec^2 a} \tag{1}$$

where K_α is the stiffness of the torsional spring, a is the lift-curve slope, q is the dynamic pressure, α is the initial angle of attack, and θ is the angle of twist.

For a given nonvanishing a, the angle θ will increase when dynamic pressure q increases. When q is so large that the denominator tends to zero, the angle θ becomes indefinitely large, and the wing is divergent. Hence, the condition of divergence is

$$K_\alpha - qec^2 a = 0 \tag{2}$$

The dynamic pressure at divergence q_{div} is given by Eq. (3) [25].

$$q_{div} = \frac{K_\alpha}{ec^2 a} \tag{3}$$

In this paper, the chord c and the torsional stiffness K_α are assumed to be uncertainties. Their bounds and other parameters of a two-dimensional wing are listed in Table 1.

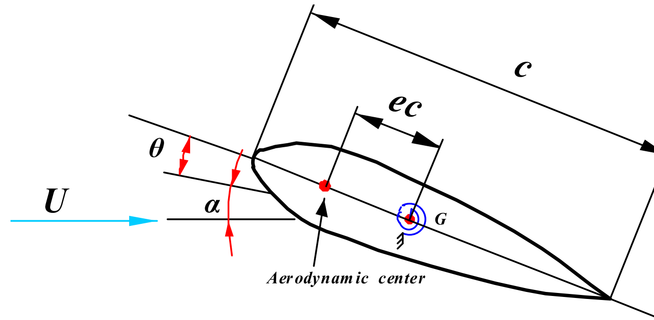

Fig. 1. A two-dimensional wing.

Table 1
Parameters of a two-dimensional wing.

Parameters	Value
c (m)	[1.35, 1.65]
K_α (N·m/rad)	[270.0, 330.0]
ec	[0.18, 0.22]
a	0.11

3. Interval analysis method of the wing divergence

The classic interval analysis (CIA), parameterized interval analysis combined with the optimization and anti-optimization problems (PIA + OAP), Taylor expansion and parameterized interval analysis combined with the optimization and anti-optimization problems (TE + PIA + OAP) are used to predict the bounds of divergence dynamic pressure of two-dimensional wing with uncertain parameters listed in Table 1, respectively. And then the results from these method are compared to demonstrate the feasibility of the method (TE + PIA + OAP) developed by this paper.

3.1. Classic interval analysis

The purpose of the classic interval analysis [22,26] is to take the uncertain parameters into consideration, and predict upper and lower bounds of response. Uncertain parameter can be described by interval variable:

$$x^I = [\underline{x}, \overline{x}] \tag{4}$$

$\underline{x}$ and $\overline{x}$ define the lower and upper bounds of interval. Alternatively an interval x^I can be represented by its central value and deviation i.e.

$$x_0 = \frac{\underline{x} + \overline{x}}{2} \tag{5}$$

$$\Delta x = \frac{\underline{x} - \overline{x}}{2} \tag{6}$$

For the interval number x^I and y^I, the basic algebraic operations are given below:

$$\begin{aligned} x^I + y^I &= [\underline{x} + \underline{y}, \overline{x} + \overline{y}] \\ x^I - y^I &= [\underline{x} - \overline{y}, \overline{x} - \underline{y}] \\ x^I \times y^I &= \left[\min(\underline{xy}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{xy}), \max(\underline{xy}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{xy})\right] \\ x^I / y^I &= [\underline{x}, \overline{x}] \times [1/\overline{y}, 1/\underline{y}] \quad \text{if } 0 \notin y^I \end{aligned} \tag{7}$$

For a two-dimensional wing example, uncertain torsional stiffness, chord and distance between the aerodynamic center and the axis of torsional spring can be represented by interval numbers, K_α^I, c^I, e^I. Due to these uncertain parameters, the divergence dynamic pressure is also uncertain, and Eq. (3) used to calculate the divergence dynamic pressure can be rewritten: