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Towards an improved critical wave groups method for the probabilistic assessment of large ship motions in irregular seas

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ABSTRACT

A novel approach for the systematic construction of wind-generated, high probability, wave groups, is presented. The derived waveforms originate from a Markov chain model allowing for the incorporation of cross-correlations between successive wave heights and periods. Analytical expressions of the transition probability distributions are provided in terms of copulas. Rank correlations are estimated from an envelope-process-based approach. The Karhunen–Loève theorem is employed in order to construct the continuous analogs of discrete height and period successions. The method seems to predict well the expected wave heights. The period predictions are conservative, yet they follow the trends of simulated wave trains. Comparisons with predictions of the “Quasi-Determinism” theory for very high runs indicate good coincidence. The derived wave groups are intended to be used for the assessment of ship stability in irregular seas.

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1. Introduction

1.1. Motivation and objective

It is well-known that the study of ship instability in a stochastic sea can easily turn into a very computationally expensive exercise. More so, if high fidelity hydrodynamic codes are employed for performing long-time simulations of ship motions and most of the time is idly expended for simulating innocuous ship-wave encounters. The efficiency of brute force computational procedures targeting the rare manifestations of ship instability is, in general, very low. Thus, a method for directly extracting those specific time intervals when dangerous wave events are realized, is highly desirable.

In this paper we present a novel approach for the systematic construction of realistic wave group profiles, characterized by a high probability of occurrence, given the sea state. The objective is the development of an efficient method for studying ship instability phenomena incurred by wave groupiness. We build further upon the so called “critical wave groups” approach, briefly reviewed next.

1.2. The “critical wave groups” concept

According to the original formulation of the “critical wave groups” approach, the probability of occurrence of a certain type of ship instability can be determined by the probability of encountering wave groups generating the instability; i.e. producing on the ship critical, or severer, excitations [1]. The principal idea is to disassemble the problem into a deterministic and a probabilistic part. In the context of the former, critical values for the key wave group characteristics (e.g., height, period and run length) are identified, from deterministic consideration of ship dynamics. The critical waveforms represent basically thresholds, defined by regular wave trains. In the probabilistic part, on the other hand, the propensity for stability failure is expressed as the probability of encountering any wave group above the determined threshold height, for a range of periods and wave group run lengths. However, defining thresholds by regular wave trains might lead to conservative conclusions.

Various other works have targeted the outcome of encounters between wave groups and ships. For example, recently, Malara et al. [2] predicted the maximum ship roll motions, in the vicinity of very high waves, by the theory of "Quasi-Determinism" [3], using the normalized autocovariance function of the wave load process.

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1.3. Advances in wave group theory

Numerous studies have been focused on the stochastic treatment of height and period successions within wave sequences. Markov chain modeling of consecutive waves has been one of the most successful approaches. The original formulation was presented by Kimura [4], who validated the model by numerical simulations. In accordance with the study of Arhan and Ezraty [5] for positive correlation between successive wave heights, Kimura [4] elaborated wave groupiness measures for sequences of discrete heights and periods that fulfill the Markov property. In his study, however, the features of wave period trains were completely independent from the related height groupings. Moreover, the correlation parameters involved in the proposed distribution laws were estimated from the simulated time-series.

Kimura's study has found keen supporters (e.g., Battjes and van Vledder [6], Sobey [7], Stansell, et al. [8]). Battjes and van Vledder [6] proposed a formula for the estimation of the correlation parameter for successive wave heights, based on Rice's theory for envelope statistics [9,10]. Van Vledder [11] proposed later an improved calculation of this parameter which was found to provide satisfactory predictions for wave group statistics in the case of sufficiently narrow-banded spectral density forms [8]. Despite the remarkable progress that has been achieved for the "wave height Markov chain", little attention has been paid, hitherto, on the corresponding wave periods. This could be attributed to the computational complications arising when a wave-envelope-based approach is used.

1.4. Key points of the current approach

Instead of elaborating on wave group statistics, the current study is focused on the construction of continuous-time wave groups whose heights and periods originate from a discrete-time Markov process. The governing equations for a Markov chain process related to the time evolution of joint wave characteristics are thus formulated. To this end, Kimura's model [4] needs to be extended, so as to incorporate cross-correlations between successive heights and periods. Moreover, continuous-time representations of this process are constructed, using the "Karhunen–Loève" theorem [12,13]. Analytic formulas for the density kernels, involved in the transition mechanisms of the Markovian system, are proposed in terms of copula probability distributions. The method is tested against Monte Carlo simulations. Comparisons with the established theory of "Quasi-Determinism" [3] are performed for cases of very high wave groups.

2. Stochastic modeling of ocean wave groups

A wave group is a sequence of waves with heights exceeding a certain preset level, H_{cr}, and periods varying within a potentially small range [14,15]. Individual waves are defined using the standard zero up-crossing method. Their height, H, is given by the maximum vertical excursion of the surface elevation between two consecutive zero up-crossings and their period, T, is defined as the time interval separating these two events. The group length j is the number of consecutive waves with heights greater than H_{cr}.

2.1. The Markov chain approach

Let us define the random vector process $Z = \{H, T\}$ which satisfies the Markov property. H is the wave height and T the associated period. $\zeta_i \in \Omega$ is the state variable of Z at time step i, where Ω is the event space of that process. The joint probability distribution of N consecutive realizations of Z is based on the following product form:

$$f_{Z_1,\ldots,Z_N}(\zeta_1, \ldots, \zeta_N) = f_{Z_1}(\zeta_1)\cdot \prod_{i=2}^{N} f_{Z_i|Z_{i-1}}(\zeta_i|\zeta_{i-1}). \tag{1}$$

For a time-homogeneous chain, the transition mechanism is the same after each time step. In terms of heights and periods it is given as:

$$f_{H_i,T_i|H_{i-1},T_{i-1}}(h_i, t_i|h_{i-1}, t_{i-1}) = \frac{f_{H_{i-1},T_{i-1},H_i,T_i}(h_{i-1}, t_{i-1}, h_i, t_i)}{f_{H_{i-1},T_{i-1}}(h_{i-1}, t_{i-1})} \tag{2}$$

The expected values of the coordinates of Z are given by:

$$\begin{aligned}\bar{h}_i &= \int_0^\infty \int_0^\infty h_i f_{H_i,T_i|H_{i-1},T_{i-1}}(h_i, t_i|h_{i-1}, t_{i-1}) dt_i dh_i \\ &= \int_0^\infty h_i f_{H_i|H_{i-1},T_{i-1}}(h_i|h_{i-1}, t_{i-1}) dh_i\end{aligned} \tag{3}$$

$$\begin{aligned}\bar{t} &= \int_0^\infty \int_0^\infty t_i f_{H_i,T_i|H_{i-1},T_{i-1}}(h_i, t_i|h_{i-1}, t_{i-1}) dh_i dt_i \\ &= \int_0^\infty t_i f_{T_i|H_{i-1},T_{i-1}}(t_i|h_{i-1}, t_{i-1}) dt_i\end{aligned} \tag{4}$$

Closed forms for the probability density functions involved in Eqs. (3) and (4), are presented in Section 3.

For a given set of initial conditions, the "most expected" sequence of wave heights and related periods can be produced using Eqs. (3) and (4) iteratively. Forward application of the iterative scheme determines the expected features of the "future" waves. For a Markov process, the time-reversibility property allows for the application of Eqs. (3) and (4) backwards in time so as to compute the expected features of the "past" waves. For practical reasons the selection of the initial conditions will be based on the features of the highest wave of the sequence. In this way, the latter is uniquely defined by the height and period of the highest wave, which occupies the center of the group.

2.2. The Karhunen–Loève representation

In order to construct the continuous-time profile of the proposed Markovian system, let us assume that the water surface elevation $\eta(t)$ is a stochastic signal defined over a fixed time interval $[-T, T]$. The Karhunen–Loève theorem [12,13] states that $\eta(t)$ accepts the following expansion:

$$\eta(t) = \sum_{n=0}^{\infty} a_n f_n(t), \quad -T < t < T. \tag{5}$$

In the case of a Gaussian random process, the coefficients, a_n ($n = 0, 1, \ldots$), are random independent variables. The computation of basis functions f_n casts in the form of a Fredholm equation of the second kind, with the kernel being the autocorrelation function R of the process:

$$\int_{-T}^{T} R(t-\tau) f_n(t) d\tau = \kappa_n f_n(t). \tag{6}$$

The κ_n parameters are the eigenvalues of the respective orthogonal functions f_n. According to Slepian and Pollack [16], there exists an explicit solution for Eq. (6) when the kernel of the integral is the *sinc* function. Sclavounos [17] employed the similarities between the latter and the typical autocorrelation form for wind generated waves, in order to compute the f_n functions. The same computational procedure is followed here (the details of the analysis are omitted for brevity).

The adjustment of the random variables a_n will be based on the predictions of Eqs. (3) and (4), for heights and periods, respectively. The key idea is to apply geometric constraints on Eq. (5) at