Probabilistic Engineering Mechanics 44 (2016) 89–98

Contents lists available at ScienceDirect

Probabilistic Engineering Mechanics

journal homepage: www.elsevier.com/locate/probengmech

Path integral solution for nonlinear systems under parametric Poissonian white noise input

A. Di Matteo [a], M. Di Paola [a], A. Pirrotta [a,b,*]

[a] Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, dei Materiali (DICAM), Università degli Studi di Palermo, Viale delle Scienze, I-90128 Palermo, Italy
[b] Department of Mathematical Sciences, University of Liverpool, Liverpool, UK

ARTICLE INFO

Article history:
Received 16 August 2015
Accepted 29 September 2015
Available online 9 October 2015

Keywords:
Nonlinear systems
Parametric input
Path integral solution
Poisson white noise

ABSTRACT

In this paper the problem of the response evaluation in terms of probability density function of nonlinear systems under parametric Poisson white noise is addressed. Specifically, extension of the Path Integral method to this kind of systems is introduced. Such systems exhibit a jump at each impulse occurrence, whose value is obtained in closed form considering two general classes of nonlinear multiplicative functions. Relying on the obtained closed form relation liking the impulses amplitude distribution and the corresponding jump response of the system, the Path Integral method is extended to deal with systems driven by parametric Poissonian white noise. Several numerical applications are performed to show the accuracy of the results and comparison with pertinent Monte Carlo simulation data assesses the reliability of the proposed procedure.

1. Introduction

Gaussian white noise processes are probably the most commonly adopted in random vibration analysis, and numerous papers have been then devoted to the study of linear and nonlinear systems under this kind of excitation. On the other hand, a different and yet very versatile model for random excitations is represented by the so-called Poisson white noise process $W_P(t)$. Samples of this stochastic process are constituted by a train of impulses of random amplitude Y occurring at random time instants distributed according to a Poisson law. The versatility of this model is simply proved considering that if the mean number of impulse occurrences per unit time $\lambda \to \infty$ and $\lambda E[Y^2]$ is constant (being $E[\cdot]$ the ensemble average operator), then $W_P(t)$ reverts to a Gaussian white noise process. Further there exist a variety of phenomena of engineering interest which are basically non-normal and may be more appropriately modeled as Poisson white noise processes. Examples are highway bridges under traffic loads [1,2], buffeting airplanes tails [3,4], vehicles or tracks traveling on rough roads [5,6] and earthquake excited structures [7–9].

The equation ruling the evolution of the probability density function (PDF) of dynamical systems under external Poisson white noise processes is the so-called Kolmogorv-Feller (KF) equation, which is an integro-differential equation with a convolution integral whose kernel is the probability of the spikes occurrences. Exact solutions for this kind of equation exist only for the stationary case of a very restricted class of systems and distribution of the impulse amplitude [10,11]. Numerical or approximate analytical procedures have been then developed to determine the evolution of the response PDF for more generic cases. Readers are referred to [12–20] and references therein for a detailed treatment of the problem.

Note that all these studies have dealt with systems under external excitations, in which the exciting forces are independent on the configuration of the systems themselves. Nevertheless, in many cases of engineering interest the exiting forces are modulated by a function of the system response, and these are usually known as *parametric* or *multiplicative*.

A comprehensive analysis on dynamical systems under random parametric excitation can be found in [21]. In this regard, it should be noted that the majority of papers pertaining this topic deal with systems under multiplicative Gaussian white noise excitation, while the relevant analysis on systems subjected to parametric Poisson white noise is much less addressed. On the other hand there are many mechanical systems in which parametric impulsive inputs arise, such as when dealing with impacts between objects and objects with a rigid barrier [22–24] or rocking motion of rigid blocks under base excitation [25–27]. Thus there is still a need of further studies on the response evaluation of systems under parametric Poisson white noise.

* Corresponding author at: Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, dei Materiali (DICAM), Università degli Studi di Palermo.
E-mail addresses: alberto.dimatteo@unipa.it (A. Di Matteo), mario.dipaola@unipa.it (M. Di Paola), antonina.pirrotta@unipa.it, Antonina.Pirrotta@liverpool.ac.uk (A. Pirrotta).

http://dx.doi.org/10.1016/j.probengmech.2015.09.020
0266-8920/

In the case of systems driven by parametric Poissonian white noise, a hierarchy of corrective terms or an infinite number of higher order derivate moments appears in the corresponding modified KF equation [28–32]. Exact solution for the stationary PDF of such cases can only be found in [11] for a very restricted class of nonlinear systems, while in [33] a perturbation procedure is considered to find the approximate stationary PDF. In [34,35] equivalent linearization and stochastic averaging techniques are adopted for multiplicative Poisson excitation, while in [36] the cumulant-neglect closure scheme has been extended to deal with these systems. In [37] a modified Monte Carlo simulation based procedure is proposed for parametrically excited systems and in [38,39] the extension of the exponential-polynomial closure method is derived to cope with Poisson white noise parametric input. Finally recently in [40] a complex fractional moments based procedure, previously developed in [20], has been applied to restitute the evolution of the response PDF of a class of nonlinear systems parametrically excited by Poisson white noise.

In this paper the Path Integral (PI) method, commonly used to determine the PDF evolution of oscillators under (external) Gaussian or Poissonian white noise [12,16–18], is extended to the case of nonlinear systems under parametric Poisson white noise excitation. This procedure is based on recent results reported in [41], for (deterministic) multiplicative impulsive input. In this regard, note that here a wide class of symmetric and antisymmetric nonlinear functions of the response process are considered to modulate the random force. Thus the response of quasi-linear system can be easily obtained as a limiting case of the corresponding antisymmetric function.

In order to assess the validity of the proposed procedure, applications to several nonlinear systems under parametric Poisson white noise are presented, and the resulting evolution of the PDF is compared with pertinent Monte Carlo simulations as well as with some benchmark solutions.

2. Jumps for parametric deterministic impulse

In this section a brief overview on some recent results related to the solution of nonlinear systems under deterministic parametric impulsive input is presented. These results will constitute the base for the following extension to systems under parametric Poisson white noise.

Consider a generic dynamical system driven by a parametric impulse, whose equation of motion is given in the form

$$\begin{cases} \dot{x}(t) = f(x,t) + \gamma_k\, g(x,t)\,\delta\left(t - t_k\right) \\ x(0) = x_0 \end{cases}; \quad t_k > 0 \tag{1.a,b}$$

where $f(x,t)$ and $g(x,t)$ are nonlinear functions of the response $x(t)$, γ_k is the amplitude of the Dirac's delta $\delta(t - t_k)$ at the time instant t_k and x_0 is the assigned initial condition. Note that if $g(x,t) = 1$ the impulse is external, otherwise the impulse is parametric. For this reason hereinafter $g(x,t)$ will be denoted as parametric (or multiplicative) function, to recall it modulates the impulsive input.

Solution of Eq. (1.a) may be obtained subdividing the time axis in three parts, that is: $t < t_k^-$, $[tk^-, tk^+]$ and $t > t_k^+$, where t_k^- and t_k^+ indicate the time instants immediately before and after the impulse respectively.

Specifically, these steps should be followed:

i. Solve the homogenous differential equation $\dot{x}(t) = f(x,t)$, $\forall\, t < t_k^-$, with initial condition x_0, and find the solution immediately before the impulse occurrence $x(t_k^-)$;
ii. Evaluate the jump $J(t_k)$ due to the Dirac's delta, so that the response immediately after the impulse is given as $x(t_k^+) = x(t_k^-) + J(t_k)$;
iii. Solve the differential equation $\dot{x}(t) = f(x,t)$, $\forall\, t > t_k^+$ assuming as initial condition the value $x(t_k^+)$.

While solution for $t < t_k^-$ and $t > t_k^+$ can be easily accomplished, particular attention should be paid to find the jump $J(t_k)$ for a generic nonlinear parametric function $g(x,t)$, unless the case $g(x,t) = 1$ is considered (external impulse) for which $J(t_k) = \gamma_k$.

In some previous papers [42] the jump for the case of parametric impulse and $g(x,t) \in C_\infty$ (the class of ∞-times differentiable functions) has been given in a series expansion

$$J\left(t_k\right) = x\left(t_k^+\right) - x\left(t_k^-\right) = \sum_{j=1}^{\infty} \gamma_k^j \frac{g^{(j)}\left(x\left(t_k^-\right), t_k\right)}{j!} \tag{2}$$

where $g^{(j)}(x,t)$ may be evaluated in recursive form as follows

$$g^{(j)}(x,t) = \frac{\partial g^{(j-1)}(x,t)}{\partial x} g^{(1)}(x,t); \quad g^{(1)}(x,t) = g(x,t) \tag{3}$$

Although Eq. (2) returns the jump for a very wide class of nonlinear parametric function $g(x,t)$, its series form is not easily manageable. On the other hand, it has been demonstrated in [41] that the jump prediction can be restituted in analytical form, once the attention is confined to some quite general form of antisymmetric and symmetric nonlinearities.

Specifically functions $g(x,t)$ of the form

$$g(x,t) = |x|^\alpha \operatorname{sgn}(x); \quad \alpha \in \mathbb{R}^+ \tag{4.a}$$

$$g(x,t) = |x|^\alpha; \quad \alpha \in \mathbb{R}^+ \tag{4.b}$$

have been considered, since they represent a wide class of nonlinearities of engineering interest. In fact solving Eq. (1.a) with $g(x,t)$ represented in Eq. (4.a), solution for the linear case $(\alpha = 1)$ as well as for nonlinearity of the type $\operatorname{sgn}(\cdot)$ $(\alpha = 0)$, cubic $(\alpha = 3)$ and so on $(|x|^{2\alpha+1}\operatorname{sgn}(x) = x^{2\alpha+1};\ \alpha = 0, 1, 3, \ldots)$ may be found. Further solving Eq. (1.a) with $g(x,t)$ expressed in Eq. (4.b), the case of external input $(\alpha = 0)$ is solved, and for $(\alpha = 2, 4, \ldots)$ the cases of the type $|x|^{2\alpha} = x^{2\alpha}$.

2.1. Antisymmetric nonlinearities

Consider the case of nonlinear parametric function in Eq. (4.a). In order to find $x(t_k^+)$ once $x(t_k^-)$ is already known, let assume that the Dirac's delta is a window function with finite duration τ and with an amplitude γ_k/τ, so that the total area of the impulse is preserved (see Fig. 1). Hence the correct value of $x(t_k^+)$ is obtained as the limit when $\tau \to 0$.

Let $z(\rho)$ be the solution of the following differential equation

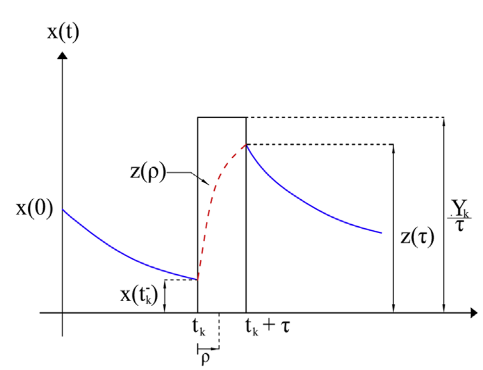

Fig. 1. Response to the window function of duration τ and amplitude Y_k/τ.