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A semi-analytical solution for free vibration of thick orthotropic annular sector plates with general boundary conditions, internal radial line and circumferential arc supports

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ABSTRACT

The aim of this paper is to put forward a semi-analytical solution to conduct the free vibration analysis of thick orthotropic annular sector plates with general boundary conditions, internal radial line and circumferential arc supports for the first time. Based on the Mindlin plate theory, stationary energy principle and Rayleigh-Ritz method, the governing eigenvalue equation is derived for thick orthotropic annular sector plates of different radii, sector angles, thickness ratio, line/arc supports and various boundary conditions along the radial and circumference edges. The main innovation point of this paper lies in breaking the classical boundary barrier by using an improved Fourier series to replace the traditional Fourier series. By comparing with those results in open reported literature, the present method proves to be of excellent accuracy and reliability. In addition, in order to enrich the research, some results involving thick orthotropic annular sector plates with various boundary conditions are presented firstly, which can be acted as benchmark data for the future research.

1. Introduction

As the fundamental components, thick orthotropic annular sector plates have been widely applied in such technological fields as naval, ocean, mechanical and civil engineering on account of their good mechanical property (Thang and Lee, 2018; Mu et al., 2015). The International Maritime Organization (IMO) states that the ships shall be constructed to reduce on board noise and to protect personnel from noise in accordance with the Code of noise on board ships. Code on noise levels has been adopted due to the recognized need to establish mandatory noise limits in different spaces of the ships. This new Code comes into force on 1st of July 2014. Much progress has to be made; especially in the topics of noise and vibration on board. The vibration performance is very important and useful for designers to satisfy the code. As one of special composite materials, the orthotropic material owing an excellent mechanical behavior results in a lot of applications in the industrial field. Thus, it is very important and useful for designers to understand the free vibration characteristic of the thick orthotropic annular sector plate with general boundary conditions, which directly relates to realizing the reasonable and precise mechanical structure design.

In the past decade, great efforts on annular sector plates have been made, and also a huge development has been obtained. By using the sector Fourier p –element, Houmat (2001) surveyed the vibration behavior of the thin sectorial plates based on the classical thin theory. Chen et al. (2004) extended the meshless method to study the free vibration of circular thin plates with clamped boundary conditions. Hajabasi and Mirtalaie (2011) applied the differential quadrature method (DQM) to perform the free vibration analysis of functionally graded thin annular sector plates with classical boundary conditions. Based on the trigonometric and exponential functions, Kim and Yoo (2010) proposed a novel analytical solution to study the flexural response of annular sector thin-plates. In framework of classical thin theory, Seok and Tiersten (2004) studied the flexural vibrations of an annular sector plate by means of a variational approximation procedure. Shi et al. (2014) used the Ritz solutions for vibration analysis of annular, circular and sector plates with arbitrary boundary conditions. Wong et al. (2000) extended the mode subtraction method to investigate the sensitivity of changes in displacement mode shape of annular plates relative to the whole size. In the above literature, the research status of thin plates has been given in detail. Next, we will give a detailed summary of the research results of thick annular sector plates. Aghdam and Mohammadi (2009) applied the extended Kantorovich method (EKM) to conduct the bending analysis of thick annular

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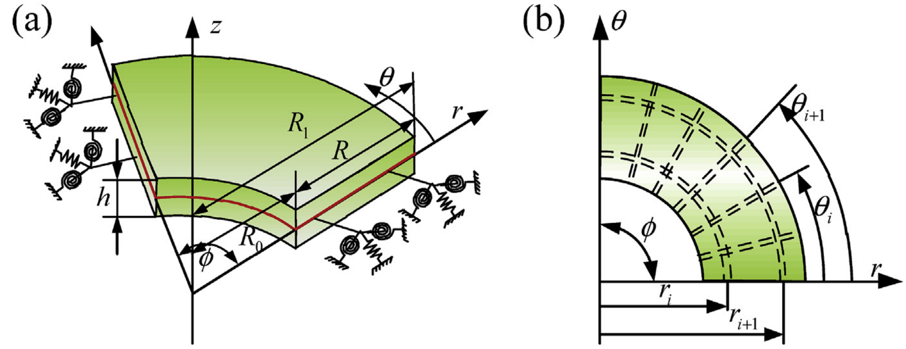

Fig. 1. Schematic diagram of a thick orthotropic annular sector plate with elastic boundary restraints: (a) geometry and dimensions of an annular sector plate; (b) partial internal line/arc supports view of the annular sector.

sector plates with classical boundary conditions based on the Mindlin plate theory. Liew et al. (1995); Liew and Liu (2000) presented the differential quadrature method (DQM) for free vibration analysis of moderately thick annular sector plates on the basis of the Mindlin first-order shear deformation theory. In their solution, the boundary conditions are limited to classical cases. McGee et al. (1995) firstly presented an exact solution to solve for the free vibration problem of thick (Mindlin) annular sectorial plates. It must be pointed out that the boundary condition is defined to be simply supported in the radial edges and arbitrary classical conditions along the circular edge of the thick plate. Mizusawa (1991) applied semi-analytical methods, which are the finite strip method and finite prism method, to study the thick annular sector plates with simply supported straight edges. Cheung and Chan (1981) extended the finite strip method to perform the analysis of thin and thick sectorial plates with variable thicknesses and different combinations of boundary conditions. Jomehzadeh and Saidi (2009a,b) proposed an analytical method to solve the free vibration problem of transversely isotropic moderately thick annular sector plates. The boundary conditions under assuming are simply supported at radial edges and arbitrary classical conditions along the circular edges. Huang and Ho (2004) put forward an analytical solution for vibration of a polarly orthotropic Mindlin circular sector plate with the radial edges simply supported. Based on the Mindlin plate theory, an exact analytical solution was presented to study the free vibration of transversely isotropic sector plates by Jomehzadeh and Saidi (2009a,b) using the boundary layer function, where the plate radial edges were simply supported and circular edges were subject to arbitrary boundary conditions. Es'haghi (2014) presented an exact closed-form solution to study the free vibration of thick circular/annular sector plates, which are with hard simply supported radial edges and subject to all combinations of free, soft simply supported, hard simply supported and clamped boundary conditions along the circular edges. In this study, the establishment of the theoretical model was according to the Reddy's third-order shear deformation plate theory. Vaidyanathan et al. (1994) apply the Rayleigh-Ritz method to determine the flexural behavior of a cantilevered annular sector Mindlin plate of variable rigidity, including the effects of the shear deformation and its boundary conditions were limited to some classical cases. Shi et al. (2017) adopted an improved Fourier series method and the first-order beam theory to study in-plane free vibration behaviors of functionally graded carbon nanotube reinforced composite (FG-CNTRC) circular arches (curved beams) under elastic boundary restraints.

As far as the authors know, there has been no specified work for the subject of free vibration of orthotropic annular sector Mindlin plates subject to general boundary conditions and with internal radial line and circumferential arc supports. Most contributions only focus on the free vibration analysis of isotropic annular sector Mindlin plates which is with classic boundary supports. Thus, the starting point of this paper is to present a semi-analytical solution served for studying the titled problem for the first time. Based on the Mindlin plate theory, an energy functional is derived and then minimized by the Rayleigh-Ritz method. The admissible displacement functions are reconstructed by using the improved Fourier series method to break the barrier of the classical boundary which is caused by the traditional Fourier series resulting in the relevant discontinuities of the traditional Fourier series and their derivatives at the edges. The convergence, correctness and accuracy of the present method is validated by several examples. For further enriching the research, some new results involving the free vibration of thick orthotropic annular sector plates are presented, where these plates are subject to different classical boundary conditions and elastic boundary conditions and internal radial line and circumferential arc supports are also added to the plate. Since there have been no related reference data so far, they may serve as reference for future researchers to verify alternative numerical methods. Besides, the effects of boundary springs, sector angle, orthotropic stiffness, radial/arc line support location on the frequency parameters of the thick orthotropic annular sector plates will be studied and then some vital and useful conclusions are obtained.

2. Theoretical and numerical formulations

As mentioned before, a flat, thick orthotropic sector plate is chosen as the study object. Just as shown in Fig. 1(a), it's with a uniform thickness of h, and its shape depends on the outer radius R_1, inner radius R_0, plate width $R(=R_1\text{-}R_0)$ in the radial direction and the sector angle ϕ. In addition, the polar coordinate system $(r,\ \theta,\ z)$ is also shown in Fig. 1(a), which will be used in the analysis. As for the internal line/arc supports internally supported in the plate, part of which is shown in Fig. 1(b), they can impose zero displacement constraint in a transverse direction.

2.1. Governing equations and boundary conditions

Each displacement component of the plate is given below according to the Mindlin plate theory:

$$u(r,\theta,z) = z\psi_r(r,\theta),\ v(r,\theta,z) = z\psi_\theta(r,\theta),\ w(r,\theta,z) = w(r,\theta) \tag{1}$$

where w is the displacement of the middle surface in z direction and ψr and $\psi\theta$ are the rotation functions. Based on the assumptions of small deformation and linear strain-displacement relations, the strain components of thick orthotropic annular sector plates are written in the following form:

$$\varepsilon_r = z\frac{\partial\psi_r}{\partial r},\ \varepsilon_\theta = z\left(\frac{\psi_r}{r} + \frac{\partial\psi_\theta}{r\partial\theta}\right),\ \varepsilon_{r\theta} = z\left(\frac{\partial\psi_r}{r\partial\theta} - \frac{\psi_\theta}{r} + \frac{\partial\psi_\theta}{\partial r}\right),\ \varepsilon_{rz} = \frac{\partial w}{\partial r} + \psi_r,\ \varepsilon_{\theta z} = \frac{1}{r}\frac{\partial w}{\partial\theta} + \psi_\theta, \tag{2}$$

According to the Hooke's law, the stress components of thick