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A discretization procedure for rare events in Bayesian networks

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ABSTRACT

Discrete Bayesian networks (BNs) can be effective for risk- and reliability assessments, in which probability estimates of (rare) failure events are frequently updated with new information. To solve such reliability problems accurately in BNs, the discretization of continuous random variables must be performed carefully. To this end, we develop an efficient discretization scheme, which is based on finding an optimal discretization for the linear approximation of the reliability problem obtained from the First-Order Reliability Method (FORM). Because the probability estimate should be accurate under all possible future information scenarios, the discretization scheme is optimized with respected to the expected posterior error. To simplify application of the method, we establish parametric formulations for efficient discretization of random variables in BNs for reliability problems based on numerical investigations. The procedure is implemented into a software prototype. Finally, it is applied to a verification example and an application example, the prediction of runway overrun of a landing aircraft.

1. Introduction

For operational risk and reliability management, it is often desirable to compute the probability of a rare event F under potentially evolving information. Examples include warning systems for natural and technical hazards, or the planning of inspection and intervention actions in infrastructure systems. Ideally, this is achieved through Bayesian updating of $\Pr(F)$ with the new information Z to the posterior probability $\Pr(F|Z)$. When physically-based or empirical models for predicting the rare event exist, such updating is possible with structural reliability methods (SRM) [24,26,31]. However, it is often difficult to perform the required computations in near-real-time, due to a lack of efficiency or robustness. A modeling and computational framework that does facilitate efficient Bayesian updating is the discrete Bayesian network (BN). Hence it was proposed to combine SRMs with discrete Bayesian networks for near-real-time computations [29,30,7].

BNs are based on directed acyclic graphs (DAGs), to efficiently define a joint probability distribution $p(\mathbf{Y})$ over a random vector $\mathbf{Y}$ [13,14]. The DAG of a BN, which is often referred to as the qualitative part of a BN, consists of a node for each variable in $\mathbf{Y}$ and a set of directed links among nodes representing dependence among the variables. In the case of discrete BNs, conditional probability tables (CPTs) quantitatively define the type and strength of the dependence among the variables. The entries of the CPT of a variable Y_i are the probabilities for each state of Y_i conditional on all possible combinations of states of its parents.

For hybrid BNs, which include both discrete and continuous variables, exact inference is available only for two special cases, which are BNs with Gaussian nodes, whose means are linear functions of their parents, and BNs, whose nodes are defined as a mixture of truncated basic functions (MoTBFs) [17,18]. Otherwise, approximate inference algorithms are available for hybrid BNs based on sampling techniques (e.g. [20,9]). However, these are computationally demanding and not generally suitable for near-real-time decision support [8]. As an alternative, the continuous random variables can be discretized, which enables the use of exact inference algorithms that exist for general discrete BNs. These include the variable elimination algorithm [32] and the junction tree algorithm [12,19].

The size of discrete BNs, and the associated computational effort, increases approximately exponentially with the number of discrete states of its nodes, which motivates the development of efficient discretization algorithms. While efficient discretization in the context of machine learning and BNs in general has been investigated by multiple researchers [15,5], research on efficient discretization in the context of engineering risk analysis or structural reliability has been limited. In general, it is to be distinguished between static and dynamic discretization. While the

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former discretizes the BN a-priori before entering evidence (offline), the latter is based on an iterative scheme that updates the discretization scheme in function of the evidence (online).

Dynamic discretization for risk analysis applications has been developed mainly by Neil et al. [21], based on the work by Kozlov and Koller [16]. The procedure starts with an initial discretization of a hybrid BN, for which an approximate entropy error is calculated. If the error complies with a convergence criterion, the current discretization is accepted. Otherwise the discretization is iteratively altered, by splitting the intervals with the highest entropy error, until the convergence criterion is fulfilled. The approach is implemented in the software AgenaRisk [1]. Other dynamic discretization algorithms for reliability analysis have been proposed, e.g. in [33] for dynamic BNs. The advantage of dynamic discretization is its flexibility when evidence is entered in the BN, i.e. when the model is updated with new observation.

Static discretization has the advantage of being computationally faster and simple to implement. Some considerations for static discretization of BNs in reliability applications have been presented in [25,29,7]. As pointed out by Friis-Hansen [7], for applications in which extreme events are important, discretization of the distribution tails should be performed with care. Static discretization facilitates a careful representation of these tails. However, the accuracy of the static discretization varies with the available evidence. The difficulty is thus to find a discretization scheme that is optimal under a wide variety of posterior distributions.

In this paper we derive a procedure for efficiently performing static discretization of continuous reliability problems. An optimal discretization scheme is sought, which minimizes the expected approximation error with respect to possible future observations (evidence). To solve this optimization problem, we propose to approximate the reliability problem by the First-Order Reliability Method (FORM). Section 2 of the paper describes the proposed methodology. Section 3 presents numerical parameter studies, and simple parametric relations for defining an efficient discretization scheme are derived. In Section 4, the procedure is applied to a set of verification examples and to the computation of the probability of runway overrun of a landing aircraft. While the theory is introduced for problems with only one design point, considerations regarding problems with multiple design points are given in the last verification example and in the discussion.

2. Methodology

2.1. Structural reliability

Since the 1970s structural reliability methods have been developed and applied in the engineering community to estimate failure probabilities $\Pr(F)$ of components or systems, based on physical or empirical models. The performance of engineering components is described by a limit state function (LSF) $g(\mathbf{x})$, where $\mathbf{X}=[X_1;\ldots;X_n]$ is a vector of basic random variables influencing the performance of the component. By definition, failure corresponds to $g(\mathbf{x})$ taking non-positive values, i.e. the failure event is $F=\{g(\mathbf{X})\le 0\}$. $g(\mathbf{x})$ includes the physical or engineering model, which is often computationally demanding. The probability of failure is calculated by integrating the probability density function (PDF) of $\mathbf{X}$, $f_{\mathbf{X}}(\mathbf{x})$, over the failure domain:

$$\Pr(F)=\int_{g(\mathbf{x})\le 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{1}$$

The formulation can be extended to the reliability of general systems by defining the failure domain as a combination of series and parallel systems [4]. In the general case, there is no analytical solution to Eq. (1) and the integral is potentially high-dimensional. For this reason, structural reliability methods (SRMs) are applied to approximate it. These include the first- and the second order reliability method (FORM and SORM) as well as a large variety of sampling methods, including importance sampling methods such as directional importance sampling, and sequential sampling methods such as subset simulation. These methods are well-documented in the literature [2,22,3,4].

2.2. First order reliability method (FORM)

To obtain an approximation of the probability of failure through FORM, the LSF $g(\mathbf{X})$ is transformed to an equivalent LSF $G(\mathbf{U})$ in the space of uncorrelated standard normal random variables $\mathbf{U}=[U_1;\ldots;U_n]$ (Fig. 1). The transformation is probability conserving, so that $\Pr[g(\mathbf{X})\le 0]=\Pr[G(\mathbf{U})\le 0]=\Pr(F)$. A suitable transformation for this purpose, which is consistent with the BN, is the Rosenblatt transformation [10]. In case all basic random variables are independent, this transformation reduces to the marginal transformations: $U_i=\Phi^{-1}\left[F_{X_i}(X_i)\right]$, with Φ^{-1} being the inverse standard normal CDF.

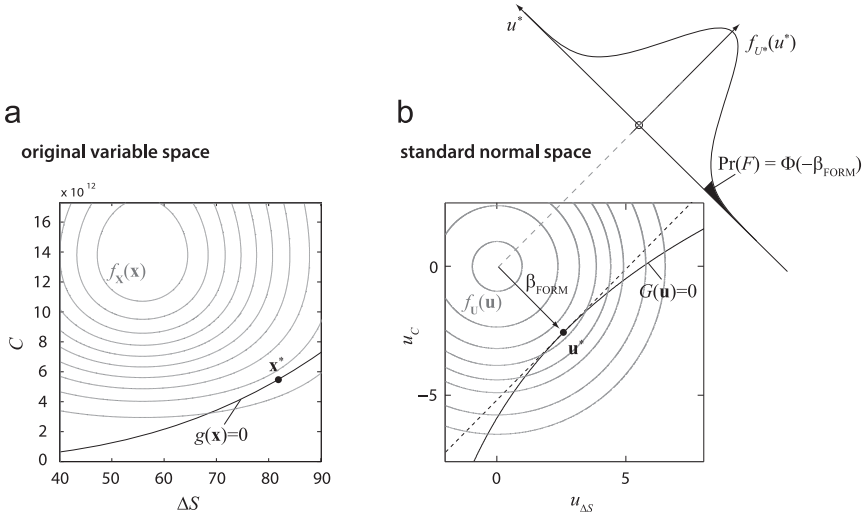

Fig. 1. Design point and linear approximation of the limit state surface. Left side: original random variable space; right side: standard normal space (from [27].