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Block replacement policy with uncertain lifetimes

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ABSTRACT

Block replacement policy is a type of preventive replacement policies where the units are always replaced at failure or at a scheduled time periodically. Assuming that the lifetimes of the units are uncertain variables, this paper studies the block replacement policy in an uncertain environment. To deal with the different constraints a decision maker may face, three different criterions are used to build the optimization models for the uncertain block replacement problem, which are chance value, expected value and critical value. Based on these criterions, the uncertain block replacement models are transformed into some equivalent crisp models. Then the optimal scheduled replacement time under each criterion is obtained and its properties are investigated.

1. Introduction

Preventive maintenance policies provide significant strategies of reducing the cost of a system which is caused by the failure and the replacement (or repair) of the units. However, it should not be carried out too frequently, otherwise the costs of preventive maintenance might outweigh the benefits. The objective of preventive maintenance is to minimize the average cost in the long run. In order to achieve this, an appropriate time for scheduled maintenance has been applied to the operations of the systems, such as the block replacement policy and the age replacement policy. This paper is dedicated to block replacement policy with uncertain lifetimes.

Block replacement, which means a unit is replaced at failure or at some scheduled time periodically, is usually used when there are a large number of similar units in a system. Assuming that the lifetimes of the units are random variables, Barlow and Proschan [3] introduced the basic model for the block replacement policy. Note that this original block replacement policy is wasteful, as a new unit might be replaced at the scheduled time. Then many modified and complex models for the block replacement policy have been designed. Barlow and Hunter [2] proposed a block replacement policy with minimal repair at failure, where the unit undergoing the repair has the same failure rate as before. Cox [7] suggested that for the scheduled replacement time T, a failed unit is replaced at once before some time $T-\delta$ or remains inactive until the next scheduled replacement. Nakagawa [17] suggested that a failed unit could be replaced by a used unit or undergo minimal repair depending on which time interval it belongs to. Sheu [22] considered the multiple choices of a unit at failure: replace it by a new or a used unit, minimally repair it, or let it remain inactive until the next scheduled replacement. Sheu and Griffith [23] considered the block replacement policy with shock models and used units as well. Recently, Sheu et al. [24] studied the block replacement policy for the system with non-homogeneous pure birth shocks. For some other applications of block replacement policy, refer to Nakagawa [18], Berthaut et al. [4], Huynh et al. [9], Khatab et al. [11], Park and Pham [21], Wang and Banjevic [27], Sheu et al. [25], Zhang et al. [31], and Nguyen et al. [19].

Recently, the maintenance strategy for multi-component systems has become a research hotspot. Anastasiadis et al. [1] constructed an aging and statistical dependence model, and derived a representation of the system's joint survival function. Chen et al. [6] developed an analytic framework for joint modeling of age-based preventive maintenance and (s, Q) spare component provisioning policy for k-out-of-n systems. Faghih-Roohi et al. [8] developed a dynamic model for the availability assessment of multi-state weighted k-out-of-n systems, and optimized the availability and capacity of the components by using genetic algorithm. Panagiotidou [20] studied a stochastic model of the joint maintenance and spare parts ordering problem, and minimized the expected total maintenance and inventory cost per time unit. Vu et al. [26] presented a dynamic maintenance grouping strategy for multi-component systems with complex structures, and optimized online the maintenance strategy in the presence of dynamic contexts. Liu et al. [16] developed a model based on renewing free-replacement warranty by considering the failure interaction among components, and presented two cost models

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for series and parallel system configurations. Zhou et al. [32] proposed a time window based preventive maintenance model for multi-component systems with the stochastic failures and the disassembly sequence, and designed a Monte-Carlo based algorithm to simulate the stochastic failures and to calculate the cumulative maintenance cost of the system.

As we know, the lifetimes of the units in a system generally involve the human uncertainty. For example, a unit under the operations of a skilled operator generally has a longer lifetime than a unit under the operations of a fresh operator. In order to deal with human uncertainty in a system, an uncertainty theory was established by Liu [12] and refined by Liu [14]. To indicate the belief degree that an uncertain event occurs, a concept of uncertain measure is proposed which satisfies the normality, duality, subadditivity and product axioms. Then an uncertain variable as a measurable function on an uncertainty space is used to represent a quantity under uncertainty, and an uncertainty distribution is used to describe an uncertain variable. In addition, concepts of expected value, variance, and entropy are also employed as some numerical characteristics.

In order to simulate the evolution of an uncertain phenomenon with the time, a concept of uncertain process was proposed by Liu [13] as a sequence of uncertain variables driven by the time. Meanwhile, the uncertain renewal process was designed to count the renewals that an uncertain system occurs. Then Liu [14] proved the elementary renewal theorem, that is, the renewal rate converges in distribution to the reciprocal of the inter-arrival time. In addition, Liu [14] proposed the uncertain renewal reward process whose inter-arrival times and rewards are modeled by uncertain variables, and Yao and Li [28] proposed uncertain alternating renewal process whose on-times and off-times are modeled by uncertain variables. As applications of the uncertain renewal processes, Yao and Ralescu [29] studied the uncertain age replacement policy, and Yao and Qin [30] studied the uncertain insurance risk process which denotes the capital of an insurance company.

Considering the human uncertainty in operating the system, this paper assumes that the lifetimes of the units are uncertain variables, and studies the uncertain block replacement policy. The rest of this paper is organized as follows. The next section introduces the uncertain block replacement policy, and by using the criterion of chance value, it transforms the uncertain block replacement problem into an optimization problem. Then by using the criterion of expected value and critical value, the uncertain block replacement problem is transformed into another two different types of optimization problems in Sections 3 and 4, respectively. Then we discuss the difference between the three models, and the difference between the uncertain block replacement policy and stochastic block replacement policy in Section 5. Finally, some conclusions are made in Section 6.

2. Chance value model

Block replacement means that the units are always replaced at failure or at some scheduled time periodically. Block replacement policy aims at finding an optimal scheduled replacement time such that the average replacement cost is minimized. In this section, we consider the human uncertainty involved in the lifetimes of the units, and introduce the uncertain variables to describe these lifetimes. Besides, we assume that the cost of replacing the failed units is larger than the cost of replacing the non-failed units. The following notations are used in the model:

$i=1,2,\cdots$: the units;
ξ_i: the lifetime of the ith unit, uncertain variable;
N_t: the uncertain renewal process with ξ_i's as the inter-arrival times;
a: the cost of replacing a failed unit;
b: the cost of replacing a non-failed unit, $b<a$;
T: the scheduled replacement time, decision variable.

According to the definition of renewal process, the units suffer N_T unexpected failures before the scheduled replacement time T. Since each failure occurs at a cost a, the cost of replacing failed units is aN_t. Noting that there is a replacement at the instant T with a cost b, we get the total replacement cost aN_T+b during the time horizon $(0,T]$. As a result, the average replacement cost is

$$\frac{aN_T+b}{T}.$$

The optimal scheduled replacement time T^* could be obtained by solving the following optimization problem:

$$\min_{T>0}\frac{aN_T+b}{T}. \tag{1}$$

However, Model (1) contains some uncertain factors, so it cannot be minimized directly. In the following sections, we will study Model (1) in three different aspects.

In some situations, we have a budget for replacing the units, and hope to minimize the chance that the average replacement cost overruns the budget. In this case, we get the chance value model of the uncertain block replacement problem:

$$\min_{T>0}\ \mathcal{M}\left\{\frac{aN_T+b}{T}>c\right\} \tag{2}$$

where c is the predetermined average replacement cost.

Theorem 1. *Let Φ denote the uncertainty distribution of the lifetimes of the units. Then the average cost $(aN_T+b)/T$ has an uncertainty distribution*

$$\Psi(x)=1-\Phi\left(T\Big/\left(\left\lfloor\frac{xT-b}{a}\right\rfloor+1\right)\right)$$

where $\lfloor x\rfloor$ denotes the maximum integer less than or equal to x.

By using the duality of uncertain measure, we have

$$\mathcal{M}\left\{\frac{aN_T+b}{T}>c\right\}=1-\mathcal{M}\left\{\frac{aN_T+b}{T}\le c\right\}=\Phi\left(T\Big/\left(\left\lfloor\frac{cT-b}{a}\right\rfloor+1\right)\right),$$

so the chance value model (2) could be transformed into

$$\min_{T>0}\Phi\left(T\Big/\left(\left\lfloor\frac{cT-b}{a}\right\rfloor+1\right)\right). \tag{3}$$

Furthermore, since the uncertainty distribution Φ is a non-decreasing function, Model (3) could be simplified into

$$\min_{T>0}T\Big/\left(\left\lfloor\frac{cT-b}{a}\right\rfloor+1\right). \tag{4}$$

Theorem 2. *Let a,b and c be some positive numbers with $a>b$. Then the optimal solution of the optimization problem (4) is*

$$T^*=\frac{b}{c}.$$

It follows from Theorem 2 that under the criterion of chance value, the optimal scheduled replacement time is $T^*=b/c$, which is independent of the cost a of replacing a failed unit. By substituting $T^*=b/c$ into Model (3), the chance that the actual replacement cost overruns the budget has a value $\Phi(b/c)$.