Ocean Engineering 154 (2018) 277–287

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

An improved model updating technique based on modal data

Kang Liu[a,b,c], Ren-Jun Yan[b], C. Guedes Soares[c,*]

[a] Key Laboratory of High Performance Ship Technology (Wuhan University of Technology), Ministry of Education, Wuhan, 430063, China
[b] School of Transportation, Wuhan University of Technology, Wuhan, 430063, China
[c] Centre for Marine Technology and Ocean Engineering (CENTEC), Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais, 1049-001, Lisboa, Portugal

ARTICLE INFO

Keywords:
Finite element model
Model updating
Damage detection
CMCM
ICMCM
Numerical rank

ABSTRACT

A novel technique of model updating named improved cross-model cross-mode (ICMCM) is presented. It is based on extracted modal data and to avoid a rank deficient problem and a biased solution of the derived linear equations, this method changes the form of coefficient matrix and adds more independent equations on the basis of the original cross-model cross-mode (CMCM) method. Truncated singular value decomposition is utilized for possible solutions. Results of two numerical investigations, including model updating of a simply-supported beam and damage detection on a jacket platform, indicate better accuracy and stability of the proposed technique. A comparative rank study reveals that this improved technique handles more unknowns with the same amount of extracted modes. Several damage cases are considered with modal data contaminated by noise. Results affected by noise level, extracted mode combinations, and different damage cases are discussed. Monte Carlo simulations show that even mode shapes contaminated with up to 3% added noise, it would still be possible to obtain satisfactory results with proper selected modes.

1. Introduction

In recent years, finite element analysis has become an essential tool for structural design. However, the fact that sometimes there is a large difference between numerical estimation and experimental measurements has demanded more accurate and reliable models. Three classes of modeling error are normally considered, i.e. parameter error, discrete error, and structural error deduced by theoretical hypothesis and other unknowns. Any of uncertainties in modeling might lead to deviation and even to wrong estimation. One general purpose of model updating is to make better correspondence between dynamic response of the analytical model and the real structure so that a more practical model could be applied for dynamic prediction, optimal design or reliability analysis. The modal parameters of structure, e.g. frequencies and mode shapes, change with modifications of physical properties. This conclusion also produces certain cases where model updating techniques are sometimes interpreted as damage detection procedure. When better correspondence of dynamic response are ensured between intact and damaged structure, the corrected physical properties from intact model could be regarded as damage indications.

The model updating technique has been developing rapidly. The general steps and problems are well elaborated in the book Finite Element Model Updating in Structural Dynamics (Friswell and Mottershead, 1995). Usually two sorts of model updating methods are considered, (i) direct matrix approach, which corrects the elements of the system matrix in a process that may destroy symmetry and physical connectivity of the original system, frequently leading to results with no physical significance. A second class is defined as (ii) iterative parameter approach, mostly based on sensitivity of the chosen dynamic response with respect to certain parameters. This approach always calls for massive calculations and convergent results are not always ensured.

Many researchers have proposed various methods using dynamic data in model updating or damage detection, such as modal strain energy (Rezaei et al., 2016; Liu et al., 2014), wavelet transform (Asgarian et al., 2016) or damping ratio (Budipriyanto et al., 2007). Several artificial intelligence algorithms, e.g. artificial neural network (Zubaydi et al., 2002), genetic algorithm (Malekzehtab and Golafshani, 2013), particle swarm optimization (Malekzehtab et al., 2012) have been introduced. Mottershead et al. (2011) gave a tutorial of model updating using sensitivity method, and probabilistic approaches (Khodaparast et al., 2008; Silva et al., 2016a,b) are applied to a set of physical structures. Literature reviews of model updating (Mottershead and Friswell, 1993), as well as of damage detection (Doebling et al., 1996) using vibration characteristics have been well elaborated.

Among many approaches, the cross-model cross-mode method (CMCM) (Hu et al., 2007) was proposed, which directly corrects stiffness

* Corresponding author.
E-mail address: c.guedes.soares@centec.tecnico.ulisboa.pt (C. Guedes Soares).

https://doi.org/10.1016/j.oceaneng.2018.02.011
Received 6 October 2017; Received in revised form 31 December 2017; Accepted 4 February 2018

and mass matrix without iterative computation, thus avoiding the convergence problem. A further attempt of simultaneously updating mass, stiffness and damping matrix was developed by Hu and Li (2007). In the CMCM method, there is no necessity for pairing or scaling of mode shapes, although the need of complete mode shape seems to be the only limitation. This drawback has been considered by some researchers (Silva et al., 2016a,b; Drozg et al., 2018). By several modal expansion and model reduction techniques, Li et al. (2008) proposed the CMCM method by taking incomplete modal data into damage detection, and Wang et al. (2015) presented an experimental study on offshore platform.

These studies have confirmed the validity of the CMCM method in few given conditions or with limited number of unknowns, but without considering the rank study of the coefficient matrix and noise effect. In other words, the stability of the solution is not always guaranteed. In essence, the CMCM method seeks to solve a system of linear equations which can lead to solutions without physical meaning if the coefficient matrix is rank deficient, as well as frequently producing biased answers especially when the extracted modal data is contaminated by noise. It is normally expected a unique solution in least-square sense with a full rank coefficient matrix while several numerical treatments, namely the regularization method, has to be conducted to correct against noise effect.

On the basis of the original CMCM method, the proposed ICMCM approach changes the coefficient matrix into a difference form, and in order to satisfy full rank condition for linear equations created by modal data, new expressions are added. Results of two numerical investigations, including model updating of a simply-supported beam and damage detection of a jacket platform, indicate better the accuracy and stability of the proposed technique. It is shown that the proposed ICMCM method provides a higher rank of the coefficient matrix with the same amount of modal data, thus larger number of unknowns could be determined in an optimized least square sense. In the second example, several damage cases are considered with modal data contaminated by noise. Results affected by the noise level, extracted mode combination, and different damage cases are discussed. Monte Carlo simulations show that even when mode shapes are contaminated with up to 3% added noise, one can still obtain satisfactory results if proper modes are selected.

In section 2, the theoretical background of the previous CMCM method and of the proposed ICMCM technique are described. Section 3 shows a brief introduction seeking for solution of linear equations, including the truncated singular value decomposition which is adopted in this paper. Two numerical applications are investigated and results are compared in section 4. Main conclusions are given in section 5.

2. Theoretical background

2.1. Original CMCM method

In the CMCM method (Hu et al., 2007), an undamped-system is assumed where the stiffness matrix and the mass matrix of the structure, denoted by $\mathbf{K}$ and $\mathbf{M}$, are extracted from the baseline FE model. The superscript $*$ denotes the parameters that stem from experimental measurements.

The expressions begin with the i th characteristic equation of the baseline FE model

$$\mathbf{K}\mathbf{\Phi}_i = \lambda_i\mathbf{M}\mathbf{\Phi}_i \tag{1}$$

and the j th characteristic equation from the experimental measurements

$$\mathbf{K}^*\mathbf{\Phi}_j^* = \lambda_j^*\mathbf{M}^*\mathbf{\Phi}_j^*, \tag{2}$$

where $(\lambda_i, \mathbf{\Phi}_i)$ and $(\lambda_j^*, \mathbf{\Phi}_j^*)$ denote the $\boldsymbol{i}$ th and $\boldsymbol{j}$ th modal pairs from the baseline FE model and experimental measurements respectively. $\mathbf{K}^*$ and $\mathbf{M}^*$ represent stiffness and mass matrix of the updated FE model respectively, which are unknown at present.

Assuming that $\mathbf{K}^*$ and $\mathbf{M}^*$ can be written in combination form with respect to all elements of the baseline FE model, one concludes that

$$\mathbf{K}^* = \mathbf{K} + \sum_{n=1}^{N_e}\alpha_n\mathbf{K}_n \tag{3}$$

and

$$\mathbf{M}^* = \mathbf{M} + \sum_{n=1}^{N_e}\beta_n\mathbf{M}_n, \tag{4}$$

where N_e is the total number of elements; $\mathbf{K}_n$ and $\mathbf{M}_n$ are stiffness and mass matrix of the n th element written in global coordinate, respectively. By definition, α_n and $\beta_{\boldsymbol{n}}$ stand for correction coefficients which are expected to be determined.

Premutiplying Eq. (1) by $(\mathbf{\Phi}_j^*)^T$ and Eq. (2) by $(\mathbf{\Phi}_i)^T$ yields

$$\left(\mathbf{\Phi}_j^*\right)^T\mathbf{K}\mathbf{\Phi}_i = \lambda_i\left(\mathbf{\Phi}_j^*\right)^T\mathbf{M}\mathbf{\Phi}_i \tag{5}$$

and

$$(\mathbf{\Phi}_i)^T\mathbf{K}^*\mathbf{\Phi}_j^* = \lambda_j^*(\mathbf{\Phi}_i)^T\mathbf{M}^*\mathbf{\Phi}_j^*. \tag{6}$$

Note that $\mathbf{K}$ and $\mathbf{M}$ are symmetric, and the transpose of a scalar stays the same, one shows that the transposition of Eq. (5) yields

$$(\mathbf{\Phi}_i)^T\mathbf{K}\mathbf{\Phi}_j^* = \lambda_i(\mathbf{\Phi}_i)^T\mathbf{M}\mathbf{\Phi}_j^*. \tag{7}$$

Dividing Eq. (6) by Eq. (7), one thus has

$$\frac{(\mathbf{\Phi}_i)^T\mathbf{K}^*\mathbf{\Phi}_j^*}{(\mathbf{\Phi}_i)^T\mathbf{K}\mathbf{\Phi}_j^*} = \frac{\lambda_j^*}{\lambda_i}\frac{(\mathbf{\Phi}_i)^T\mathbf{M}^*\mathbf{\Phi}_j^*}{(\mathbf{\Phi}_i)^T\mathbf{M}\mathbf{\Phi}_j^*}. \tag{8}$$

Substituting Eqs. (3) and (4) into Eq. (8) results in

$$1 + \sum_{n=1}^{N_e}\alpha_n C_{n,ij}^\dagger = \frac{\lambda_j^*}{\lambda_i}\left(1 + \sum_{n=1}^{N_e}\beta_n D_{n,ij}^\dagger\right), \tag{9}$$

where

$$C_{n,ij}^\dagger = \frac{(\mathbf{\Phi}_i)^T\mathbf{K}_n\mathbf{\Phi}_j^*}{(\mathbf{\Phi}_i)^T\mathbf{K}\mathbf{\Phi}_j^*} \tag{10}$$

and

$$D_{n,ij}^\dagger = \frac{(\mathbf{\Phi}_i)^T\mathbf{M}_n\mathbf{\Phi}_j^*}{(\mathbf{\Phi}_i)^T\mathbf{M}\mathbf{\Phi}_j^*}. \tag{11}$$

Rearranging and replacing ij with a new index m, Eq. (9) becomes

$$\sum_{n=1}^{N_e}\alpha_n C_{n,\mathrm{m}}^\dagger - b_m^\dagger\sum_{n=1}^{N_e}\beta_n D_{n,m}^\dagger = b_m^\dagger - 1 \tag{12}$$

or

$$\sum_{n=1}^{N_e}\alpha_n C_{n,\mathrm{m}}^\dagger + \sum_{n=1}^{N_e}\beta_n E_{n,m}^\dagger = f_m^\dagger, \tag{13}$$

where $b_m^\dagger = \frac{\lambda_j^*}{\lambda_i}$, $E_{n,m}^\dagger = -b_m^\dagger D_{n,m}^\dagger$ and $f_m^\dagger = b_m^\dagger - 1$.

The symbol $\dagger$, represents the term in equations that consist of the i th component from the FE model along with the j th component from the experimental measurements. When the first N_i th numbers of modes are obtained from the FE model, and the first N_j th numbers of modes are extracted from the experimental measurements, a total number of $N_m = N_i \times N_j$ equations can be formulated from Eq. (13). Rewriting those equations in matrix form, one gets