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# A unified model for analyzing free vibration and buckling of end-bearing piles

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ABSTRACT

Offshore structures are often founded on long, slender piles that extend for a substantial distance above the ground surface. This paper presents a novel unified model to analyze the free vibration and buckling of partially embedded end-bearing piles subjected to axial compressive load. Consideration is given to the tapered piles of variable cross-sectional shape with constant volume. The governing differential equation of the motions is derived, and solved by using the Runge-Kutta method in combination with the Regula-Flasi method. The accuracy of the proposed model is confirmed by comparing the obtained calculations with existing closed-form and numerical solutions. Numerical results for the natural frequency, buckling load and corresponding modal displacements are provided, which are analyzed to highlight the effects of the parameters related to the cross-sectional shape, taper ratio and embedment of the pile, soil stiffness and compressive force as well as the end constraint. The geometry and material parameters that statically and dynamically yield the strongest piles with fixed volume are identified. The analytical model is beneficial for the optimum design of the soil-pile system in engineering applications.

## 1. Introduction

Long, slender piles that extend above the ground are widely used for offshore structures such as causeways, cross-sea bridges, wind turbines and jacket platforms. Stability and free vibration analyses of the pile foundations are inherent parts of the design, in order to prevent collapse or severe damage of the pile-supported structures due to static load and resonance.

Extensive literature exists concerning the vibration behavior of beam-columns without soil medium, and many studies have been devoted to modeling the free vibration of beam-columns on elastic foundation (e.g., Chen, 2002; Balkaya et al., 2009; Li et al., 2012). However, a few studies have been performed to characterize the free vibration of end-bearing piles (or axially loaded beam-columns in a Winkler foundation). Ragab and Aggour (1986) examined the effect of lumped mass on the fundamental frequency of a fully embedded pile. Valsangkar and Pradhanang (1987) presented the closed-form solution for predicting the natural frequency of partially supported piles. Catal (2002) explored the influence of shear deformation on the free vibration of partially embedded piles. Yesilce and Catal (2008) dealt with the free vibration of the piles embedded in two-layered soil. Yesilce (2011) used the differential transform method (DTM) and differential quadrature element method (DQEM) for analyzing the free vibration of fully embedded Reddy-Bickford piles.

Over the past many years, several studies have been undertaken to investigate the buckling response of end-bearing piles. For example, Bjerrum (1957) addressed the theoretical buckling load of a fully embedded hinged-hinged pile. Davisson and his colleague (1963, 1965) formulated the governing differential equations for estimating the buckling of fully and partially embedded piles and reported the numerical solutions for various combinations of end conditions. Lee (1968) validated the solutions of Davisson and Robinson (1965) by comparing the results from physical modeling. Prakash (1987) employed the energy method to develop an analytical model for the buckling capacity of fully embedded piles. West et al. (1997) interpreted the buckling load and mode shape of fully embedded piles and identified the phenomenon of modal clustering. Shields (2007) suggested a semi-empirical formulation for pile buckling loads and pointed out that with the ongoing evolution of pile applications to include higher pile capacities, the common assumption that buckling does not occur for fully embedded piles is no longer valid. By accounting for material non-linearity, Vogt et al. (2009) studied the buckling of fully embedded piles in soft soil. Catal (2014) used the DTM to determine the buckling load of evaluating semi-rigid connected and partially embedded piles. Based on the cusp catastrophe theory,

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Chen et al. (2015) obtained the buckling load of fully embedded piles.

The objective of this study is to present a unified model for estimating free vibration and buckling of partially embedded end-bearing piles under axial compressive load. Here the volume of the pile is always held constant, although the geometry of the pile is dependent on the variation in the shape and area of cross section along the axis of the pile. The governing equation of the problem is derived and solved numerically. The versatility of the analytical model is illustrated using numerical examples of the soil-pile system for a wide range of geometry and material properties. The computed results are compared with closed-from and numerical solutions available.

## 2. Mathematical formulation

Fig. 1(a) shows a partially supported homogenous pile with total length $l$ and embedded length $l_e$ subjected to a compressive axial load $P$ at the top end of the pile. In Cartesian coordinate system $(x, y)$, the origin is taken to be at the bottom end of the pile. The pile has a regular polygon cross section with variable depth $d$, defined as the distance between the centroid and vertex, throughout its length. The volume of the pile is taken to be constant but the upper and lower cross section of the pile is changed. Linear variation is considered for the depth of the pile as

$$F = (r-1)\frac{x}{l} + 1 \text{ for } 0 \le x \le l \tag{1}$$

where, $r$ is the taper ratio, defined as

$$r = \frac{d_t}{d_b} \tag{2}$$

in which, $d_t$ and $d_b$ are the cross-sectional depths at top and bottom ends, respectively. It is noted that the pile is tapered up with $x$ direction for $0 < r < 1$ and tapered down for $r > 1$. Obviously, the case of prismatic pile corresponds to $r = 1$. The cross-sectional depth $d$, projection depth $w$, area $A$ and moment of inertia $I$ of the cross section of the pile are given by

$$d = d_b F \tag{3}$$

$$w = c_1 d_b F;\; c_1 = 1 + \cos\left(\frac{\pi}{2m}\right) \tag{4}$$

$$A = c_2 d_b^2 F^2;\; c_2 = \frac{m}{2}\sin\left(\frac{2\pi}{m}\right) \tag{5}$$

$$I = c_3 d_b^4 F^4;\; c_3 = \frac{c_2}{12}\cos^2\left(\frac{\pi}{m}\right)\left[3 + \tan^2\left(\frac{\pi}{m}\right)\right] \tag{6}$$

where, $m$ is the positive integer ($\ge 3$), which means the side number of the regular polygon cross section. From Eqs. (4)–(6), it is clear that when $m$ reaches infinity, the values of $c_1$, $c_2$ and $c_3$ are 2, $\pi$ and $\pi/4$, respectively, i.e., solid circular cross section. The volume $V$ of the pile can be expressed as

$$V = \int_0^l A dx = c_2 c_4 d_b^2 l;\; c_4 = \frac{1}{3}\left(r^2 + r + 1\right) \tag{7}$$

Inserting $d_b$ in Eq. (7) into Eqs. (3)–(6) yields

$$d = \sqrt{\frac{V}{c_2 c_4 l}} F; \tag{8}$$

$$w = c_1 \sqrt{\frac{V}{c_2 c_4 l}} F; \tag{9}$$

$$A = \frac{V}{c_4 l} F^2; \tag{10}$$

$$I = c_3 \left(\frac{V}{c_2 c_4 l}\right)^2 F^4 \tag{11}$$

For the embedded portion of the pile, as shown in Fig. 1(a), the lateral stiffness of the soil is modeled with an elastic Winkler foundation with a constant coefficient of subgrade reaction $K$ with unit of force per length$^3$. This assumption is commonly used for cohesive soil (Prakash and Sharma, 1990; Randolph and Gourvenec, 2011). The soil reaction $R_S$ is proportional to the lateral deflection $y$ of the pile and is formed as

$$R_S = Kwy \tag{12}$$

For a harmonic vibration mode, the inertial force against the deflection are supposed by

$$F_I = \rho A \omega_i^2 y \tag{13}$$

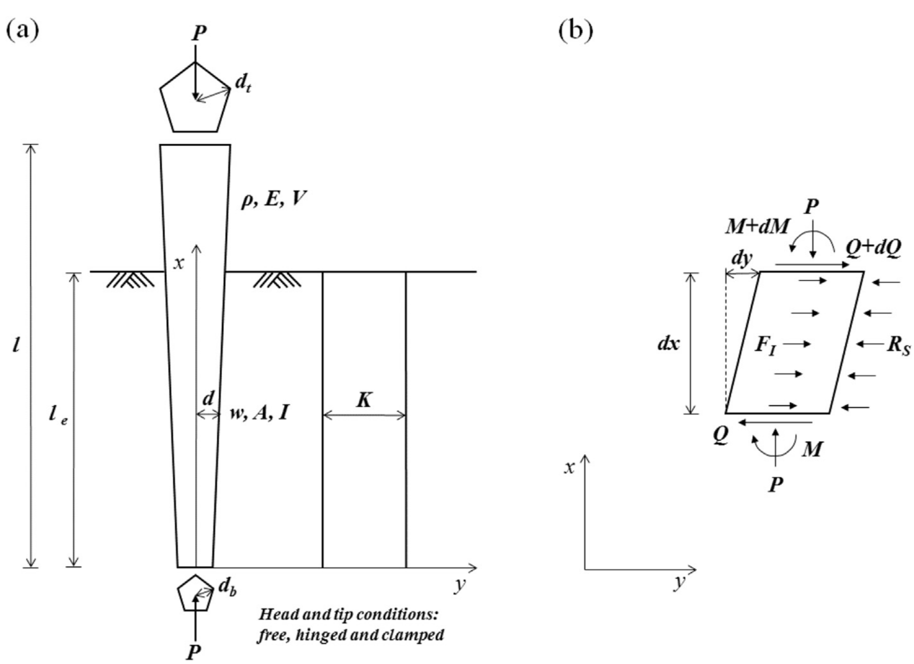


Fig. 1. Definition sketch of proposed model: (a) a partially embedded tapered pile with constant volume in the Cartesian coordinate system; (b) deformation and forces on differential pile element.