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A general boundary-fitted 3D non-hydrostatic model for nonlinear focusing wave groups

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A B S T R A C T

This paper employs a three-dimensional (3D) non-hydrostatic model to simulate nonlinear focusing wave groups. The non-hydrostatic model utilizes an explicit projection method to solve the Navier–Stokes equations. To accurately simulate the steep free surface involved in focusing waves, the model is built upon a general boundary-fitted coordinate system. This grid system allows for a great adaptability of the vertical discretization and meanwhile maintains the boundary-fitted properties of better fitting the bed and free surface. The advantage of the general boundary-fitted model is first validated by two test cases of nonlinear waves, including nonlinear standing waves and two-dimensional (2D) focusing freak wave. Then, the model is applied to simulate 2D focusing waves in deep and intermediate-water depths and 3D focusing waves in deep-water depth. By comparing with experimental data, the model results well reproduce the main characteristics of 2D deep-water focusing waves and 2D intermediate-water focusing waves as well as 3D deep-water focusing waves, demonstrating the model's capability to resolve 2D or 3D focusing wave groups. Furthermore, in the test of 2D intermediate-water focusing waves, the downstream shifting of the focusing position and time is also studied numerically, which is not presented in the experiments.

1. Introduction

Whether in open seas or coastal areas, wave group focusing is one of the physical mechanisms contributing to the formation of freak water waves, which have destructive effects on marine structures. Many physical experiments (Baldock and Swan, 1996; Baldock et al., 1996; Brown and Jensen, 2001; Johannessen and Swan, 2001; Kit et al., 2000; Ma et al., 2010; Onorato et al., 2006; Toffoli et al., 2010) have been carried out to study the focusing process based on deterministic or statistical methods. In parallel with the experimental studies, efforts have been focused on establishing numerical models in the hope that more details of focusing wave groups may be obtained. In the past few decades, potential flow models (Ducrozet et al., 2012; Fochesato et al., 2007; Toffoli et al., 2010; Yan and Ma, 2008) and nonlinear Schrödinger-type equations models (Chiang et al., 2007; Henderson et al., 1999; Osborne et al., 2000; Toffoli et al., 2010) are widely used to study focusing wave groups. The potential flow equation is derived from the incompressible Navier–Stokes equations (NSE) under the assumption of perfect fluid and irrotational motion, while the nonlinear Schrödinger-type equations can be viewed as the deviation from the potential flow equation. In contrast to non-hydrostatic models based on the NSE, they are computationally very efficient, especially for the nonlinear Schrödinger-type equations models. However, with the increase in computational power, non-hydrostatic models for the simulation of focusing wave groups are getting more attention (Young and Wu, 2010; Young et al., 2007). Most importantly, in contrast to aforementioned potential flow models and nonlinear Schrödinger-type equations models, non-hydrostatic models have the potential for simulations of freak waves resulting from various mechanisms, such as geometrical focusing, wave–current interaction, atmospheric forcing, modulation instability, etc. For more details on these different mechanisms see the reviews by Kharif and Pelinovsky (2003).

To develop numerical models based on the NSE, the treatment of free surface is one of the main issues. Many well-known methods to simulate this moving boundary have been successfully incorporated in the NSE, such as the volumes of fluid method (VOF), the level set method and the smoothed particle hydrodynamics method (SPH). All of these methods are capable of dealing with complicated free surfaces (e.g. overturning waves), but their applications mainly focus on two-dimension (2D) problems because of high computational expense. Cui et al. (2012)

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employed an improved VOF model to investigate effects of the uneven bottom topography on 2D freak waves. Dao et al. (2011) developed a SPH model enhanced with parallel computing to reproduce well 2D freak waves and their breaking process. In contrast to these models, the so-called non-hydrostatic models employ a more efficient method that tracks the free surface motion using a single-valued function of the horizontal plane. With such a method capturing the free surface, non-hydrostatic models can predict accurately a range of short wave motions, where wave shoaling, nonlinearity, dispersion, refraction, and diffraction phenomena occur. For more details about this, the reader is referred to Ai et al. (2011), Badiei et al. (2008), Stelling and Zijlema (2003), Yuan and Wu (2006), Young et al. (2009) and Zijlema and Stelling (2005). Moreover, with momentum conservative properties non-hydrostatic models can accurately predict wave breaking and run-up in the surf zone (Ai and Jin, 2012; Zijlema and Stelling, 2008; Zijlema et al., 2011) and by coupling a turbulence model, it is also capable of simulating complex wave-structure interactions (Ai and Jin, 2010; Li and Lin, 2001). However, to accurately simulate nonlinear focusing wave groups is still challenging for non-hydrostatic models.

The spatial–temporal focusing of wave groups at one point in space and time produces steep water waves, which results in one of the difficulties on developing a boundary-fitted non-hydrostatic model. It is well-known that in simulating steep free surface flows or flows over steep bottom topography, traditional boundary-fitted models (e.g. σ-coordinate models) are inaccurate because large errors may arise in the discretization of pressure gradient term. To reduce the errors of boundary-fitted models, there are many methods, such as implementing higher-order discretization to estimate the pressure gradient term (Beckman and Haidvogel, 1993; McCalpin, 1994), transforming the boundary-fitted grid back to a Cartesian system (Slordal, 1997) and using a general boundary-fitted grid system (Deleersnijder and Beckers, 1992; Decoene and Gerbeau, 2009; Shchepetkin and McWilliams, 2005), etc. The aforementioned methods have been successfully implemented in the ocean model, which mainly focuses on the simulation of stratified free surface flows over steep bottom topography. Recently, Young et al. (2007) developed a higher-order σ-coordinate non-hydrostatic model, in which a higher-order finite difference scheme is applied to discretize the horizontal pressure gradient term. Their model well predicts nonlinear surface waves (Young et al., 2007) and focusing wave groups (Young and Wu, 2010) with steep free surfaces. Unfortunately, this model is restricted to 2D problems and it is difficult to extend it to a three-dimensional (3D) model because of the implementation of higher-order spatial discretization. In this paper, we will develop a 3D non-hydrostatic model based on a general boundary-fitted grid system. It has been demonstrated that a model with the general boundary-fitted grid system can effectively reduce errors in the discretization of pressure gradient in the simulation of stratified free surface flows over steep bottom topography (Decoene and Gerbeau, 2009). Here, we will present that the general boundary-fitted non-hydrostatic model also can accurately resolve steep surface waves.

For 3D focusing wave simulations, the main restriction of non-hydrostatic models is the computational efficiency. For the numerical solution of 3D non-hydrostatic models, almost all of the computational time is spent in resolving the Poisson equation, since the overall efficiency of the numerical code will depend on its performance. In this study, the general boundary-fitted non-hydrostatic model is developed based on the former model (Ai et al., 2011), which adopted a new grid arrangement to construct a symmetric and positive definite Poisson equation. Therefore, the model is computationally very efficient by using the preconditioned conjugate gradient method to solve the Poisson equation.

The remainder of this paper is organized as follows. Section 2 presents the governing equations and boundary conditions. The numerical algorithms used in solving the non-hydrostatic model and the general boundary-fitted grid system are described in Section 3. Validations of the advantage of the general boundary-fitted grid system are presented in Section 4. Model applications to focusing wave groups and conclusions are provided in Sections 5 and 6, respectively.

2. Governing equations and boundary conditions

2.1. Governing equations

Non-hydrostatic free surface flows are governed by the 3D incompressible Navier–Stokes equations, which are expressed in the following form, by splitting the pressure into hydrostatic and non-hydrostatic ones, $p = g(\eta - z) + q$:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -g\frac{\partial \eta}{\partial x} - \frac{\partial q}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -g\frac{\partial \eta}{\partial y} - \frac{\partial q}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{3}$$

$$\frac{\partial w}{\partial t} + \frac{\partial uw}{\partial x} + \frac{\partial vw}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial q}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

where t is the time; u, v and w are the velocity components in the horizontal x, y, and vertical z directions, respectively; p is the normalized pressure divided by a constant reference density; η is the free surface elevation; q is the non-hydrostatic pressure component; g is the gravitational acceleration; ν is the kinematic viscosity.

2.2. Boundary conditions

Boundary conditions are required at all the boundaries of a 3D domain including the free surface and the bottom. At the moving free surface and the impermeable bottom, kinematic boundary conditions are applied. To calculate the moving surface, the following free surface equation is used, which is obtained by integrating the continuity Eq. (1) over the water depth and applying the kinematic free surface condition and bottom condition:

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\int_{-h}^{\eta} u dz + \frac{\partial}{\partial y}\int_{-h}^{\eta} v dz = 0 \tag{5}$$

where $-h(x, y)$ is the bottom surface.

At the inflow boundary, in order to generate focusing waves, the normal velocity components are specified as follows.

For 2D (or unidirectional) focusing waves, following Young et al. (2007), the imposed normal velocity components can be written as

$$u(x, z, t) = \sum_{n=1}^{N}\left\{a_n\omega_n\frac{\cosh\,[k_n(h+z)]}{\sinh(k_n h)}\cos\,[k_n(x - x_f) - \omega_n(t - t_f)]\right\} \tag{6}$$

where N is the number of frequency components; a_n defines the amplitude of each component; and k_n and ω_n denote the wavenumber and frequency of each component, respectively, satisfying the linear dispersion relationship. In addition, x_f and t_f are the theoretical focusing position and time, respectively.

For 3D (or directional) focusing waves, the wave group is defined by a specified range of wave components having the required spread in both frequency and direction. The imposed