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Importance of parametric uncertainty in predicting probability distributions for burst wait-times in fissile systems

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ABSTRACT

A method of uncertainty quantification in the calculation of wait-time probability distributions in delayed supercritical systems is presented. The method is based on Monte Carlo uncertainty quantification and makes use of the computationally efficient gamma distribution method for prediction of the wait-time probability distribution. The range of accuracy of the gamma distribution method is examined and parameterised based on the rate and magnitude of the reactivity insertion, the strength of the intrinsic neutron source and the prompt neutron lifetime. The saddlepoint method for inverting the generating function and a Monte Carlo simulation are used as benchmarks against which the accuracy of the gamma distribution method is determined. Finally, uncertainty quantification is applied to models of the Y-12 accident and experiments of Authier et al. (2014) on the Caliban reactor.

1. Introduction

In a fissile system, the fluctuations in neutron population over time are driven by the branching of neutrons chains, an inherently random process. When the neutron population is large, the Law of Large Numbers determines that the outcome of the many branching processes taking place in the system, will tend towards the average (or "expected") behaviour; the likelihood of significant deviations from the expected behaviour is small and can often be neglected. It is well known, however, that the behaviour of a fissile system when the neutron population is small, such as a reactor start-up in the presence of a weak neutron source, cannot be accurately modelled without considering the stochastic nature of the growth in these neutron chains. In these cases, significant deviations from the average behaviour are to be expected.

1.1. Relative strength of an intrinsic neutron source

A useful qualitative indication of the relative strength of a given neutron source was derived by Hansen (1960) who noted that a source should be considered weak if,

$$\frac{2S\tau}{\bar{\nu}\Gamma_2} \ll 1, \quad (1)$$

where S is the neutron source strength in n/s, τ is the prompt neutron lifetime, $\bar{\nu}$ is the average number of neutrons released per fission, and $\Gamma_2 = \overline{\nu(\nu-1)}/\bar{\nu}^2$. This expression is approximately equivalent to the more simple inequality,

$$S\tau \ll 1. \quad (2)$$

1.2. The wait-time

The implications of a low neutron population are well documented and have been demonstrated experimentally, by Hansen (1960) and Authier et al. (2014), amongst others. In both examples, a fast burst reactor, GODIVA (Hansen) or Caliban (Authier et al.), was brought multiple times from a subcritical state to a delayed supercritical state, and the time taken to reach a pre-defined fission rate threshold was measured. The time taken to reach the threshold is known as the *wait-time* and it was shown to vary significantly between each realisation of the experiment, despite identical experimental conditions.

In a delayed supercritical system, the neutron population at any given moment consists of prompt neutrons emitted from fission and delayed neutrons emitted from those fission fragments which are delayed neutron precursors. In a delayed supercritical system, the population of delayed neutron precursors increases over time. This leads to an increase in the number of prompt neutron chains initiated and a corresponding increase in the fission rate and the

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prompt neutron population. The wait-time is the time taken for this build up in the neutron population to occur. It varies because the growth rate in the neutron population depends on the different events which can happen to each neutron emitted in the system (absorption, scattering, leakage, etc.) and these events are inherently random. The sequence of events can never be identical for two realisations of the same experiment and this can be observed on the macroscopic scale as variation in the wait-time.

The wait-time is an important concept that can have significant implications for the severity of accidental power excursions and for the safe start-up of nuclear reactors. During reactor startup, for example, control rods may be withdrawn to increase the reactivity of the system. Assuming the reactor behaves in a deterministic manner, the reactor power will begin to increase as soon as the system reactivity increases, and once the system is critical, an exponential increase in power should be observed. This will cause a rise in temperature, leading to negative reactivity feedback through material expansion and Doppler broadening, limiting the overall reactivity of the system and preventing an excessive increase in power. However, if the neutron source is too weak, or the withdrawal of the control rods too fast, there is likely to be a delay between reaching positive reactivity and any significant rise in reactor power. Meanwhile the reactivity of the system continues to rise in the absence of any negative temperature feedback. When the power output does finally begin to rise, the system reactivity may already be large enough to produce a dangerous power excursion. One potential objective in seeking to characterise the wait-time is to know at what rate the control rod can be withdrawn so the probability of a dangerous power excursion remains below a specified *safety probability*, e.g. 10^{-8}.

1.3. Methods for predicting the wait-time probability distribution

Methods for determining the wait-time probability distribution include Hansen's method (Hansen, 1960), the Fourier series method (see Abate and Whitt, 1992) and the saddlepoint method of Hurwitz et al. (1963). Hansen's method is approximative and based on neutron survival probabilities. Hansen's method considers that the wait-time consists of two parts: the time taken before a persistent neutron chain is initiated and the time for the neutron population due to the first persistent chain to build up to the wait-time threshold. Persistent chains sponsored after the first are not considered to influence the wait-time significantly, because prompt neutron chains grow very rapidly; and unless the delay between the initiation of the first and second persistent chains were on the order of the generation time, the neutron population due to the second chain would be insignificant compared to that due to the first. Hansen notes that the initiation of a second persistent chain on this timescale is unlikely in a weak source scenario.

The Fourier series and saddlepoint methods rely on inversion of probability generating functions to obtain the probability distribution of the neutron population. This approach is more rigorous but also far more computationally expensive. These methods do not rely on the concept of the first persistent chain and are therefore able to account for the possibility of overlapping chains (initiated by different source neutrons) contributing to the neutron population at the moment the wait-time threshold is exceeded. This is important in delayed supercritical systems, where persistent fission chains consist of finite prompt chains linked by delayed neutron precursors. Since the delayed neutron precursors decay on a long timescale, compared to the generation time, overlapping fission chains become a significant possibility. The generating function methods can be applied to point models or space-dependent models. They can also be used with single or multiple energy groups.

A less expensive alternative to the generating function approach is to approximate the wait-time probability distribution using the gamma distribution method. This method was first proposed by Harris (1964) and relies on the fact that the neutron population in a multiplying system will tend towards a gamma distribution. There are cases when the neutron population does not conform to the gamma distribution so this method does not work for all scenarios, however it can be highly accurate and fast when applied to certain problems. This method has been applied by Williams (2016) to the Caliban experiments, with close agreement between the predicted wait-time probabilities and the experimental results of Authier et al. (2014) – see Section 1.5.

1.4. Types of uncertainty

For the purposes of the discussion that follows, it will be useful to distinguish between **aleatoric uncertainty** and **parametric uncertainty**. Aleatoric uncertainty will hereafter refer to uncertainty resulting from the random, stochastic nature of the build-up of neutron chains in fissile systems and parametric uncertainty will refer to that resulting from epistemic uncertainty in the input parameters. When calculating the wait-time probability distribution using any method, there will inevitably be some epistemic uncertainty in the input parameters, particularly when simulating accidental excursions where the exact chain of events may be unknown. The uncertainty in input parameters adds to the uncertainty already present due to aleatoric uncertainty.

The objective of this paper is to demonstrate a method for quantifying the impact of epistemic uncertainty in the input parameters. Uncertainty quantification (UQ) will be carried out using the Monte Carlo approach, which requires a fast method for determining the wait-time probability distribution, so that many calculations can be performed for a range of randomised sets of input parameters. The gamma distribution method will be used for this purpose, with verification of the predicted probability distribution using the saddlepoint method.

The ability to quantify the impact of epistemic uncertainty in the input parameters on the resulting wait-time probability distribution has important implications for criticality safety and safe reactor start-up.

1.5. Purpose of the current work

The method of uncertainty quantification presented in this paper makes use of the gamma distribution using the method presented in Williams (2016). In his paper, Williams shows that the wait-time probability distributions observed during the experiments of Authier et al. (2014) on the Caliban reactor, can be accurately predicted using the gamma distribution method. Starting from the forward form of the generating function equation, Williams derives equations for the mean neutron population, precursor group populations, detector counts and their corresponding covariances. These same equations will be used in this paper as the basis for the gamma distribution method.

The purpose of the current work is to incorporate the gamma distribution method demonstrated by Williams into a Monte Carlo algorithm, for the purpose of uncertainty quantification. The gamma distribution method for determining the wait-time probability distribution is particularly amenable to Monte Carlo uncertainty quantification due to its excellent computational efficiency compared to alternative, more rigorous methods.

The gamma distribution method has been shown to produce accurate results for certain scenarios, however it will be shown in Section 2.3 of this paper that the neutron population does not always conform to a gamma distribution. The present work will examine the parameters influencing the accuracy of the gamma distribution method in order to establish the range of transients to which this method of uncertainty quantification can be applied.