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A theoretical model for the vibration of fuel rod with multi spans supported by springs

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ABSTRACT

The prediction of fuel rod vibration is of importance in the designing of reactor core. A precise simulation of the vibration amplitude of fuel rods is necessary. It is disappointed that the mode shape equation of a spring-supported fuel rod with multi spans is not available in previous literature, due to the singularity of coefficient matrix. In this work, a theoretical model for the vibration of spring-supported rod with multi spans is established. The fuel rod is assumed to be a homogeneous beam. The flexural displacement of the rod is represented as the combination of Fourier series and auxiliary trigonometric functions in order to overcome the discontinuity and singularity of mode shape. The vibration frequency and mode shape are obtained. The calculation results are in satisfactory with theoretical results. The variation of spring constant and its effect could be simulated.

1. Introduction

Fuel rods are the most important components in nuclear reactor. Due to the high flow velocity of reactor coolant, the vibration and fretting wear could be found on the surface of fuel rods surrounding the spacer grid. This problem of fretting wear is not caused by fluid elastic instability which causes excessive vibration and failure in short time but because of turbulence-induced vibration that generates small amplitude (Paidoussis, 1981). The turbulence-induced vibration may cause long-term fretting-wear damage. Based on the ultrasonic tests, almost one hundred KOFA fuel rods (FRs) were found to be leaking due to the grid-to-rod fretting wear (Kim, 2010). Therefore, the fretting wear caused by turbulence-induced vibration is a major concern for the design and operation of nuclear fuel rod assemblies in pressurized water reactors (PWRs).

Much Research has been conducted for understanding the involved phenomena and causes responsible for turbulence-induced vibration and fretting wear of fuel rods. Kang et al. (2003) proposed an axial flow-induced vibration model for a single span rod supported by two springs at both ends, on the basis of a one-mode approximation. The natural frequency and mode shape functions for the flow-induced vibration were derived with Lagrange's method. Several experiments were carried out to analyze the effect on the wear of coolant flow induced vibrations and pressure fluctuations (Choi et al., 2004; Lee and Kim, 2013). In these experiments, the gaps between the springs and the fuel rod were pre-set to account for the effect of irradiation-induced contact relaxation and cladding creep at the end of life. Jiang et al. (2016) investigated the hydraulic flow-induced impact intensity between the fuel rods and spacer grids with finite element method. They developed three-dimensional models, with detailed geometries of dimples and springs of the actual spacer grids, for flow impact simulation. Liu et al. (2017) investigated the interaction of fuel rods in a spacer grid with mixing vane subjected to strong turbulent flow with commercial software. The flow-induced vibration of fuel rods and the surrounding components were analyzed. It was found that the vibration was contributed by wall shear forces and pressure.

The spacer grid has several springs, what we called springs and dimples, in a cell in order to support the fuel rod flexibly. Therefore, the supporting method for the fuel rod is actually not a simple support but a spring-support. It was reported that the spring constants of the springs and dimples gave a significant effect to the modal parameters of the FR (Kang et al., 2003). Meanwhile, the supporting spring will lose its supporting force due to the irradiation and fuel rod creep down. In these processes, the spring constant will be decreases. The conventional theoretical models (Chen, 2017) treated the fuel rod as simple-supported beam. This assumption is not suitable for the fuel rod supported with irradiated soft springs. Therefore the theoretical models for the vibration of spring-supported fuel rod are necessary in the prediction of turbulence induced vibration and fretting wear of fuel rod.

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Nomenclature

a	vector
A	area, coefficient
c	coefficient
C	vector
E	Young's modulus
H	four order full rank matrix
I	moment of inertia
K	coefficient matrix
L	span length
m	external moment
p	external force, auxiliary function
P	coefficient matrix
Q	coefficient matrix
t	time
x	coordinate
Y	mode shape function
Greek letters	
φ	initial phase
ρ	density
τ	coefficients
ω	frequency
ξ	trigonometric function
Subscripts	
k	1, 2, 3, 4
n	nth mode shape
s	span number

The most difficult thing in predicting the vibration of a spring-supported rod is to get the eigenvector (mode shape) equation. It is known that there is no method so far to get the exact solution for the eigenvector of the rod supported by two translational springs at both ends, due to the coupling effect between the translational modes of the springs and the beam modes of the rod (Kang et al., 2003). Because of that, the rod is usually assumed to be simple-supported beam for simplicity in the prediction of fuel rod vibration.

In this work, a theoretical model for the vibration of spring-supported fuel rod is established, on the basis of the work of Johansson et al. (2013) and Zhang (2015). The method proposed by Johansson et al. (2013) could result in singular matrix due to the hyperbolic form of mode shape. The method of Zhang (2015) is not exactly applicable for the fuel rod supported by springs. Because of these two shortcomings, these two methods are partly introduced in this work. The rod displacement is represented as the combination of Fourier series and auxiliary trigonometric functions to overcome the discontinuity and singularity of mode shape. The springs and dimples within a grid were simplified as a spring. Then the frequency and mode shape of the vibration of fuel rod are obtained. The variation of spring constant and its effect on the vibration amplitude of fuel rod could be simulated. This model which takes the effects of supporting springs into account is more reasonable compared with the simple-supported model for the fuel rod vibration.

2. Theoretical models

In reactor core, a fuel rod is supported by several springs and dimples, as shown in Fig. 1. According to the targeted spacer grid design, there are two dimples on one side of a slot and one spring on the opposite side (Guangdong Nuclear Power Training Center, 2004). The distances between these springs and dimples in one spacer grid are very small. Thus the springs and dimples within one grid could be simplified as a single spring, as shown in Fig. 2. Then the theoretical models for the vibration of fuel rod could be established.

The vibration of fuel rod supported by springs could be expressed as:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\rho A\frac{\partial^2 y}{\partial t^2}=p(x,t)-\frac{\partial}{\partial x}m(x,t) \tag{1}$$

where E is the Young's modulus of elasticity, I is the moment of inertia, ρ is the density of fuel rod, A is the cross-sectional area of the rod, t is time, x is the coordinate along the rod, p and m are external force and moment, respectively.

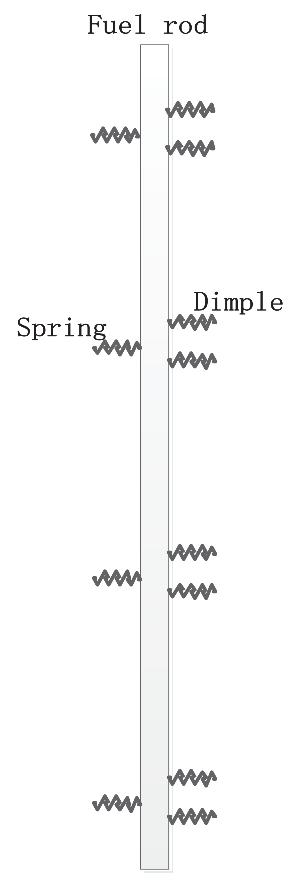

Fig. 1. Schematic of a fuel rod supported by springs and dimples.

Then the equation for the free vibration of fuel rod could be obtained as:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\rho A\frac{\partial^2 y}{\partial t^2}=0 \tag{2}$$

The solution of Eq. (2) could be written in the form of

$$y(x,t)=Y(x)\sin(\omega t+\varphi) \tag{3}$$

where ω is the angular frequency. φ is the initial phase. The mode shape function $Y(x)$ was usually expressed as the sum of trigonometric functions and hyperbolic functions (Song, 2000), which results in serious tedious problem in beams supported by springs. In this work, the mode shape function is expressed as: