ARTICLE IN PRESS

Annals of Nuclear Energy xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

Higher moments of the distribution of detector counts in a subcritical reactor

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ARTICLE INFO

Article history:
Received 8 January 2015
Accepted 6 February 2015
Available online xxxx

Keywords:
Stochastic processes
Neutron noise
Variance-to-mean ratio
Skewness
Kurtosis
Negative binomial distribution

ABSTRACT

Neutron noise methods are well established and known techniques for the experimental analysis and surveillance of nuclear reactors. One of these methods, the variance to mean ratio or α-Feynman, is based on the measurement of the histogram of the number of counts of a detector in a subcritical reactor driven by a external neutron source. The method uses the first and second moments of the distribution of counts. As a further step in the full characterization of the distribution we compute, in addition, third and fourth moments showing the limitations of approximated counts distributions.

1. Introduction

Good summaries on the neutron noise problem as well as bibliographical data can be found in the work of Pacilio et al. (1976), in the book of M. M. R. Williams (1974) and in the proceedings on the NATO conference on the subject (Muñoz-Cobo and Difilippo Edts, 1989). In their work Pacilio et al. discussed the different approximations for the probability generating function in order to calculate the moments of the histogram of the number of counts in a time interval t. One in particular is the negative binomial distribution (NBD). This distribution was used later by Difilippo (2002) to analyze how to correct the measured histograms of the number of counts due to dead-time effects. All these developments address only the first and second moments of the distributions of counts. We add further moments to characterize the distribution, including statistical parameters as the skewness and the kurtosis.

2. Equations for the moments of the distribution of counts

In this section we discussed how to generate a set of differential equations for the factorial m moment of the distribution of counts r, $< r(r-1)(r-2)\ldots(r-m+1) >$, in a measuring interval t. For random effects without correlation the distribution is the Poisson one; due to the correlation introduced via the fission process the variance (σ^2) divided the mean ($\bar{r}$) is not equal to one. Its departure from one $\sigma^2/\bar{r} \equiv 1+\psi(t)$ is used to measure the dynamic parameters of the subcritical system.

2.1. Probability distribution function

Defining $P(n,r,t)$ as the probability of having at time t, n neutrons in the system and r counts in our detector a difference-differential equation, or probability balance equation, can be written for $P(n,r,t)$ provided we have Markovian processes defined in the following way: $S\Delta t$ is the probability for the emission of one neutron by the source in the interval Δt, similarly $\Lambda_f\Delta t$, $\Lambda_c\Delta t$, and $\Lambda_d\Delta t$, are the probabilities per neutron to produce, respectively, a fission, a capture or a detection events.

Rather than trying to solve the difficult difference-differential equation for $P(n,r,t)$ it is easier to deal with the probability distribution function (PDF) defined as

$$F(x,y,t) = \sum_{n=0}^{\infty}\sum_{r=0}^{\infty} x^n y^r P(n,r,t) \quad (1)$$

and to compute the factorial moments from the partial derivatives of $F(x,r,t)$ evaluated at $x=y=1$.

From the probability balance equation it can be demonstrated that $F(x,r,t)$ satisfies the equation,

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http://dx.doi.org/10.1016/j.anucene.2015.02.008
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$$\frac{\partial F}{\partial t} = S(1-x) + \left[\Lambda_c(1-x) + \Lambda_f(p(x)-x) + \Lambda_d(y-x)\right]\frac{\partial F}{\partial x} \quad (2)$$

in this equation

$$p(x) = \sum_{\nu_p=0}^{\infty} x^{\nu_p} p_{\nu_p} \quad (3)$$

where p_{ν_p} is the probability of the emission of ν_p prompt neutrons in the fission process. Eq. (2) corresponds to the prompt neutron approximation which in general is a good one provided that time interval t is small in comparison with the decay times of the delayed precursors. Any moment of the distribution $P(n,r,t)$ can be calculated with the factorial moments which are solutions of a system of ordinary differential equations. More explicitly, the $(m+i)$ partial derivative of F with respect to x (m times) and with respect to y (i times) evaluated with Eq. (2) at $x = y = 1$ is

$$\left(\frac{\partial^{(m+i)}}{\partial x^m \partial y^i}\frac{\partial F}{\partial t}\right)_{x=y=1} = \langle n(n-1)(n-2)\ldots.(n-m+1)r(r-1) \times (r-2)\ldots.(r-i+1)\rangle \quad (4)$$

where the bracket indicate the average with the distribution *P(n,r,t)*.

2.2. System of differential equations up to the fourth factorial moments

The partial derivatives of $\partial F/\partial t$ in Eq. (4) are calculated according to the right hand side of Eq. (2). For the first moments we have

$$\frac{d<n>}{dt} = S + \alpha <n> \quad (5.1)$$

and

$$\frac{d<r>}{dt} = \Lambda_d <n>, \quad (5.2)$$

where the prompt decay constant $\alpha = -\Lambda_a + \bar{\nu}_p\Lambda_f$, with $\Lambda_a = \Lambda_c + \Lambda_f + \Lambda_d$ and $\vec{\nu}_p$ the average number of prompt neutrons per fission.

Equations for the second factorial moments are

$$\frac{d<n(n-1)>}{dt} = 2\alpha <n(n-1)> + (2S + \Lambda_f <\nu_p(\nu_p-1)>) <n> \quad (6.1)$$

$$\frac{d<nr>}{dt} = \alpha <nr> + S <r> + \Lambda_d <n(n-1)> \quad (6.2)$$

and

$$\frac{d<r(r-1)>}{dt} = 2\Lambda_d <nr> \quad (6.3)$$

where $<\nu_p(\nu_p-1)> = \sum_{\nu=0}^{\infty}\nu_p(\nu_p-1)p_{\nu_p}$.

Equations for the third factorial moments are

$$\frac{d<n(n-1)(n-2)>}{dt} = 3\alpha <n(n-1)(n-2)> + 3(S + \Lambda_f <\nu_p(\nu_p-1>) <n(n-1)> + <\nu_p(\nu_p-1)(\nu_p-2)><n> \quad (7.1)$$

$$\frac{d<n(n-1)r>}{dt} = 2\alpha <n(n-1)r> + (2S + \Lambda_f <\nu_p(\nu_p-1)>) <nr> + \Lambda_d <n(n-1)(n-2)> \quad (7.2)$$

$$\frac{d<nr(r-1)>}{dt} = \alpha <nr(r-1)> + 2\Lambda_d <n(n-1)r> + S <r(r-1)> \quad (7.3)$$

and

$$\frac{d<r(r-1)(r-2)>}{dt} = 3\Lambda_d <nr(r-1)> \quad (7.4)$$

Equations for the fourth factorial moments are

$$\frac{d<n(n-1)(n-2)(n-3)>}{dt} = 4\alpha <n(n-1)(n-2)(n-3)> + (4S + 6\Lambda_f <\nu_p(\nu_p-1>) <n(n-1)(n-2)> + 4\Lambda_f <\nu_p(\nu_p-1)(\nu_p-2)>) <n(n-1)> + \Lambda_f <\nu_p(\nu_p-1)(\nu_p-2)(\nu_p-3))><n> \quad (8.1)$$

$$\frac{d<n(n-1)(n-2)r>}{dt} = \Lambda_d <n(n-1)(n-2)(n-3)> + 3S <n(n-1)r> + 3\alpha <n(n-1)(n-2)r> + \Lambda_f <\nu_p(\nu_p-1)(\nu_p-2)><nr> + 3\Lambda_f <\nu_p(\nu_p-1)><n(n-1)r> \quad (8.2)$$

$$\frac{d<n(n-1)r(r-1)>}{dt} = (\Lambda_d + \alpha) <n(n-1)r(r-1)> + 2\Lambda_d <n(n-1)(n-2)r> + 2S <nr(r-1)> \quad (8.3)$$

$$\frac{d<nr(r-1)(r-2)>}{dt} = \alpha <nr(r-1)(r-2)> + 3\Lambda_d <n(n-1)r(r-1)> + S <r(r-1)(r-2)> \quad (8.4)$$

and

$$\frac{d<r(r-1)(r-2)(r-3)>}{dt} = 4\Lambda_d <nr(r-1)(r-2)> \quad (8.5)$$

2.3. Solutions for the factorial moments equation

There are fourteen coupled first order ordinary differential equations to calculate the factorial moments of the distribution of counts up to the fourth order. Fortunately: (1) the equations are linear with constant coefficients and (2) they can be solved sequentially in the written order, i.e. step i is the input to step $i + 1$. Each equation is of the form

$$\frac{dy}{dt} + a(t)y = f(t) \quad (9)$$

with general solution (Pontryagin, 1962)

$$y(t) = y(0)e^{-g(t)} + e^{-g(t)}\int_0^t f(\tau)e^{g(\tau)}d\tau \quad (10)$$

where $g(t) = \int_0^t a(\tau)d\tau$.

The case of the steady state of a subcritical reactor with an external source S is of interest because of standard noise techniques. Long after the introduction of the source (in terms of $1/\alpha$) the steady values of $<n>, <n(n-1)>, <n(n-1)(n-2)>$ and $<n(n-1)(n-2)(n-3)>$ can be calculated by Eqs. (5.1), (6.1), (7.1) and (8.1) by equating to zero the time derivatives. Once these variables are calculated the different moments that include the number of counts r can be calculated analytically with the use of Eq. (10), providing zero initial conditions for the moments that include r, i.e. our detector system is reset each time we perform a measurement.

In this way we found in the literature the solution up to the second moment. The second central moment (the variance $\sigma^2 = <r^2> - <r>^2 = <r(r-1)> + <r> - <r>^2$) is related to the mean value $<r>$ in the following way