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Letter

The finite deformation of the balloon catheter

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HIGHLIGHTS

- We developed an analytical model to determine the shape of inflatable catheter.
- The devices integrated on the catheter can be located by the mechanics model.
- The latitudinal elongation is much larger than the longitudinal elongation of inflated catheter.

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ABSTRACT

The balloon-based catheters are attractive for the minimally invasive procedures because these catheters can be configured to match requirements on size and shape for the interaction with the soft tissue. An analytical mechanic model is developed for the deformed balloon to determine the shape of the inflated catheter. The bridges along latitudinal direction should be high stretchable due to the high elongation along the latitude of the inflatable catheter. These results agree well with the finite element method without any parameter fitting.

Catheters are widely-used surgical tools for minimally invasive procedures to improve human health [1–6]. Devices and sensors are integrated on catheters to establish biocompatible interfaces between the semiconductor devices and the soft, curvilinear surfaces of the body. Balloon-based catheters are designed as heterogeneous collections of minimally-invasive medical devices which are integrated on the deformable skin of catheters. As shown in Fig. 1, the balloon-based catheters can be stretched up to 200% [7]. The limitation of the integration of electronic system is overcome to integrate devices and sensors on the catheter due to the high stretchability of the catheter skin. Consequently, a key point of the balloon-based catheter is to determine the locations of the different devices and the elongation between the devices while the catheters are inflated, which is necessary for the use of catheters during procedures.

As shown in Fig. 2(a), the deflated catheter is cylindrical, whose length and radius are $2L$ and R_0, respectively. After air is blown into catheter, the catheter skin will be expanded to a balloon, whose maximum radius is R shown in Fig. 2(b). The inner layer of catheter is almost undeformed since the inner layer is much thicker than the catheter skin, and the length of inflated catheter is fixed as $2L$ during the catheter skin expanding, which cross section can be described as an ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{R^2} = 1, \quad (1)$$

where a, R are the lengths of the semi-major axis and semi-mini axis, respectively. The coordinates x, y are along the directions of the semi-major axis and semi-mini axis. The length and initial radius of deflated catheter satisfy the relationship, $\frac{L^2}{a^2} + \frac{R_0^2}{R^2} = 1$. The angle, θ, between the x axis and the tangent direction of the ellipse is given by

$$\theta = \arccos\left\{\left[1 + \frac{R^2}{L^2}\left(1 - \frac{R_0^2}{R^2}\right)\left(\frac{L^2}{x^2}\frac{R^2}{R^2 - R_0^2} - 1\right)^{-1}\right]^{-\frac{1}{2}}\right\}. \quad (2)$$

The arc length of the ellipse, ds, is

$$ds = dx\sqrt{1 + \frac{R^2}{L^2}\left(1 - \frac{R_0^2}{R^2}\right)^2\left[\frac{L^2}{x^2} - \left(1 - \frac{R_0^2}{R^2}\right)\right]^{-1}}, \quad (3)$$

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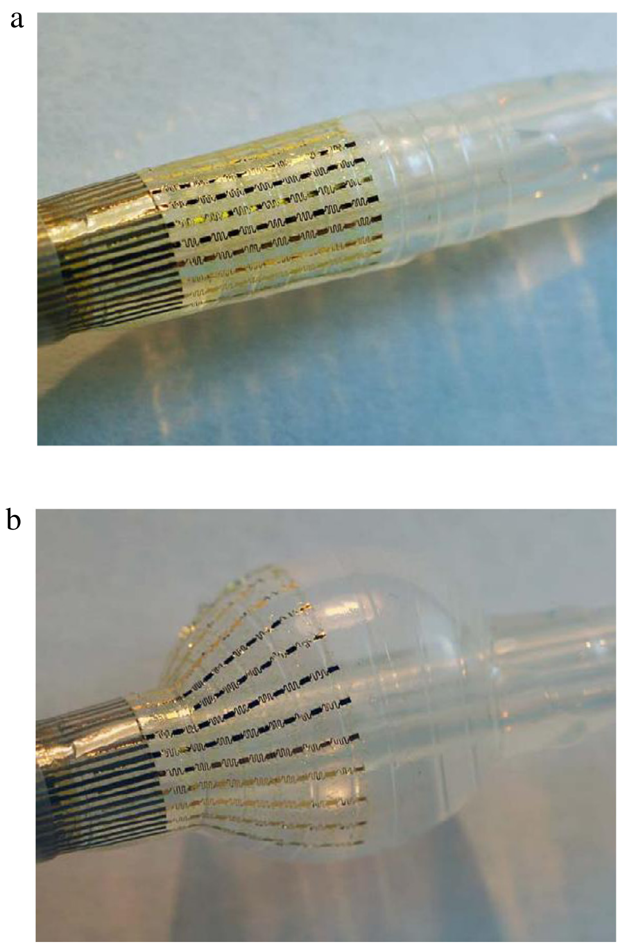

Fig. 1. Multifunctional inflatable balloon-based catheter. (a) Optical image of a stretchable, interconnected mesh integrated on a deflated catheter; (b) optical image of the inflated balloon catheter.

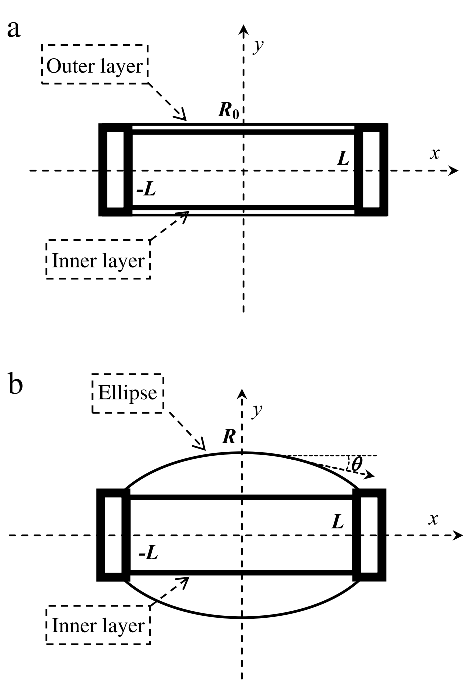

Fig. 2. Schematic diagrams of the balloon catheters. (a) Deflated catheter with outer and inner layers; (b) inflated catheter with ellipse and undeformed inner layer.

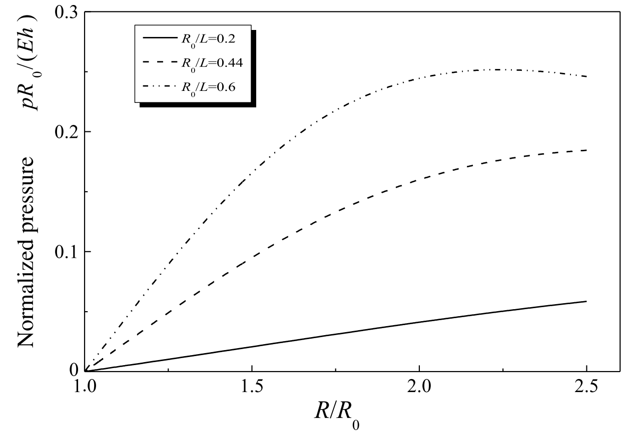

Fig. 3. The normalized pressure, $\frac{pR_0}{Eh}$, versus the ratio, R/R_0, for $R_0/L = 0.2$, 0.44, and 0.6.

which is deformed from the initial length $\mathrm{d}X$ of the cylindrical shell. The relationship between $\mathrm{d}s$ and $\mathrm{d}X$ can be given by the elongation ε along the tangent direction of the ellipse as

$$\mathrm{d}X = \frac{\mathrm{d}s}{1+\varepsilon}. \tag{4}$$

The equilibrium equation along x axial direction can be obtained from an ellipsoidal shell [8] as $t\cos\theta + \int_x^L p\mathrm{d}y = 0$, where p, t are the pressure of blown air and the stress along the tangent direction of balloon catheter, respectively. The stress can be derived from the above equilibrium equation as

$$t = pR_0\left(\frac{R}{R_0}\sqrt{1-\left(1-\frac{R_0^2}{R^2}\right)\frac{x^2}{L^2}}-1\right) \times \left[1+\left(1-\frac{R_0^2}{R^2}\right)\frac{R^2}{L^2}\left(\frac{R^2}{R^2-R_0^2}\frac{L^2}{x^2}-1\right)^{-1}\right]^{\frac{1}{2}}. \tag{5}$$

The stress is related to the elongation by the constitutive relationship, $t = Eh\varepsilon$, where E, h are the elastic modulus and thickness of the catheter skin.

The initial length X can be given as in Box I.

Because the normalized pressure satisfied the condition, $\frac{pR_0}{Eh} \ll 1$, Eq. (6) can be expanded as the Taylor series of the normalized pressure, $\frac{pR_0}{Eh}$, as

$$X = \int_0^x \left[1+\frac{R^2-R_0^2}{L^2}\left(\frac{R^2}{R^2-R_0^2}\frac{L^2}{\xi^2}-1\right)^{-1}\right]^{\frac{1}{2}}\mathrm{d}\xi - \frac{pR_0}{Eh}\int_0^x\left[1+\frac{R^2-R_0^2}{L^2}\left(\frac{R^2}{R^2-R_0^2}\frac{L^2}{\xi^2}-1\right)^{-1}\right] \times \left(\frac{R}{R_0}\sqrt{1-\frac{\xi^2}{L^2}\frac{R^2-R_0^2}{R^2}}-1\right)\mathrm{d}\xi + O\left[\left(\frac{pR_0}{Eh}\right)^2\right]. \tag{7}$$

The normalized pressure, $\frac{pR_0}{Eh}$, can be obtained from the condition of the fixed-length of the inflated balloon catheter ($X = L$ at $x = L$) as in Box II where $E\left[\sqrt{1-\left(\frac{R^2}{R_0^2}-1\right)\frac{R_0^2}{L^2}}, \arcsin\left(\sqrt{1-\frac{R_0^2}{R^2}}\right)\right]$ is the incomplete elliptic integral of the second kind. Figure 3 shows that the normalized pressure is almost linearly dependent on the ratio $\frac{R}{R_0}$ while the ratio $\frac{R_0}{L}$ is small.