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Letter

Acoustomechanics of semicrystalline polymers

Fengxian Xin [a,b,c,*], Tianjian Lu [a,c]

[a] State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an 710049, China

[b] School of Engineering and Applied Sciences, Harvard University, MA 02138, USA

[c] MOE Key Laboratory for Multifunctional Materials and Structures, Xi'an Jiaotong University, Xi'an 710049, China

HIGHLIGHTS

- An acoustomechanical theory for semicrystalline polymers is established.
- We demonstrate that acoustic radiation force is capable of causing giant deformation in these materials.
- We demonstrate that pull-in instability can be acoustically triggered even if the in-plane mechanical force is fixed.
- The findings of this study enable reliability design of novel acoustic actuated devices.

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ABSTRACT

We develop an acoustomechanical theory for semicrystalline polymers and demonstrate that acoustic radiation force is capable of causing giant deformation in these materials. When a polymer layer is subjected to combined tensile mechanical force in plane and acoustic force (sound pressure) through thickness, it becomes initially homogeneously thin but soon inhomogeneous when the two forces reach critical conditions. Critical conditions for such acoustomechanical instability are theoretically determined based on the J_2-deformation theory. We demonstrate that pull-in instability can be acoustically triggered even if the in-plane mechanical force is fixed. Bifurcation in the critical condition for acoustomechanical instability occurs when the polymer exhibits sufficiently large hardening. The findings of this study enable reliability design of novel acoustic actuated devices.

Semicrystalline polymers are extensively applied in electromechanical devices, due to their superior performance on large mechanical actuation induced by external stimuli such as electric/magnetic field, temperature, and mechanical stress [1–3]. For such applications, failures like necking and pull-in instability should be avoided, since excessive thinning down of polymer films often happens when actuated by external stimuli. As previous studies mainly focused upon electrical actuation and electromechanical instability of semicrystalline polymers, there is a basic lack of understanding of their acoustic actuation and acoustomechanical instability. Innovative design of acoustic actuated devices urgently calls for theoretical guideline on acoustomechanical behavior of semicrystalline polymers.

This paper aims to investigate the acoustic actuation and acoustomechanical instability of semicrystalline polymers subjected to combined mechanical force and ultrasound pressure. The focus is placed upon critical conditions for mechanically induced necking instability at prescribed acoustic inputs and acoustically induced pull-in instability at prescribed mechanical forces.

As illustrated in Fig. 1, we consider a thin layer of semicrystalline polymer having thickness H and in-plane dimensions $L \times L$ in undeformed state. When the polymer is subjected to equal biaxial mechanical forces in plane and two opposing ultrasound wave inputs along thickness direction, it deforms to current state with dimensions $(H\lambda, L/\sqrt{\lambda}, L/\sqrt{\lambda})$. The polymer is assumed nearly incompressible, which can support acoustic wave penetration. The constraint of incompressibility simplifies significantly subsequent theoretical analysis. This assumption generally holds true since the volume change of semicrystalline polymer is much smaller than its shape change, even at large deformation.

As the propagation of ultrasound wave in a semicrystalline polymer is accompanied by acoustical momentum transfer between adjacent medium particles, a steady time-averaged acoustic

* Corresponding author at: State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi'an Jiaotong University, Xi'an 710049, China.
E-mail addresses: fengxian.xin@gmail.com (F.X. Xin), tjlu@mail.xjtu.edu.cn (T.J. Lu).

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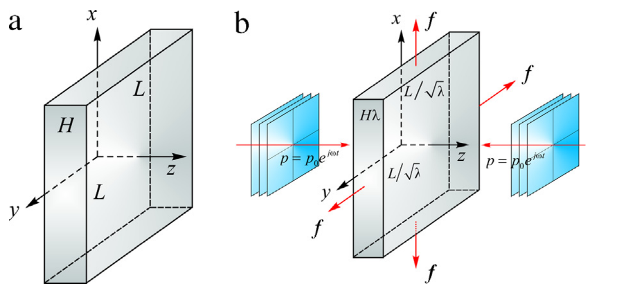

Fig. 1. (Color online) (a) A thin layer of semicrystalline polymer in undeformed state with dimensions (H, L, L); (b) deformation of polymer under acoustic loads and equal biaxial mechanical forces in current state with dimensions ($H\lambda$, $L/\sqrt{\lambda}$, $L/\sqrt{\lambda}$).

radiation stress is generated along wave propagation path in the medium, which can be expressed as [4–8]

$$\langle T_{ij}\rangle = \left[\frac{\langle p^2\rangle}{2\rho_a c_a^2} - \frac{\rho_a \langle u_k \cdot u_k\rangle}{2}\right]\delta_{ij} + \rho_a \langle u_i \cdot u_j\rangle, \tag{1}$$

where T_{ij} is the momentum flux tensor, δ_{ij} is the Kronecker delta, ρ_a is the medium density, c_a is the acoustic speed at equilibrium state, p is the acoustic pressure, and u_i is the medium velocity in the i-direction. Note that when excited by acoustic input having high enough acoustic frequencies (e.g., $\geq 10^6$ Hz, far exceeding common mechanical frequencies), deformation of a material is governed by acoustic radiation stress, which is simply the mean momentum flux tensor $\langle T_{ij}\rangle = (\omega/2\pi)\int_0^{2\pi/\omega} T_{ij}\mathrm{d}t$, ω being angular frequency. The magnitude of typical focused acoustic pressure falls within the range of 0.1–4 MPa. Correspondingly, the acoustic radiation stress scaled as $p_0^2/\left(2\rho_a c_a^2\right)$ can reach 70 247–112 MPa in air, which is sufficient to cause large deformation of semicrystalline polymers since these materials often have an elastic modulus around mega Pascal [9–12].

Accounting for both the nonlinear elastic behavior of semicrystalline polymers and acoustic radiation stress, we write the Cauchy stress in the material as

$$\sigma_{ij} = F_{iK}\frac{\partial W(\boldsymbol{F})}{\partial F_{jK}} - \left[\frac{\langle p^2\rangle}{2\rho_a c_a^2} - \frac{\rho_a \langle u_k \cdot u_k\rangle}{2}\right]\delta_{ij} - \rho_a \langle u_i \cdot u_j\rangle, \tag{2}$$

where $F_{iK} = \partial x_i/\partial X_K$ is the deformation gradient and $W(\boldsymbol{F})$ the Helmholtz free energy function. We adopt the J_2-deformation theory [13] to consider the nonlinear deformation, obtaining thence $W(\boldsymbol{F}) = K(\ln\lambda)^{N+1}/(N+1)$ for which the parameters K and N can be obtained from experimental stress versus strain curves. K scales with polymer yield strength, thus much smaller than polymer elastic modulus. N describes polymer strain-hardening. $N = K = 1$ corresponds to linear elasticity while $N = 0$ corresponds to ideal plasticity. For semicrystalline polymers, N typically varies between 0.1 and 0.6 [14,15].

Since the semicrystalline polymer is taken as nearly incompressible, its deformation state under equal biaxial forces and acoustic inputs will remain almost unchanged if a hydrostatic stress is superimposed. Therefore, the deformation state under the combined loads is approximately equivalent to that under uniaxial compressive stressing, as illustrated in Fig. 2. Following the J_2-deformation theory, the true logarithmic strain is expressed as $\varepsilon = \ln\lambda$ when extension and $\varepsilon = -\ln\lambda$ when compression. Therefore, nonlinear large deformation of the semicrystalline polymer can be described as

$$\sigma + (t_1 - t_3) = K(-\ln\lambda)^N, \tag{3}$$

where the Cauchy stress $\sigma = f/\left(HL\sqrt{\lambda}\right)$, the acoustic radiation stress $t_1 = 1/(H\lambda)\int_0^{H\lambda}\langle T_{11}(z)\rangle\mathrm{d}z$ and $t_3 = \langle T_{33}^{\text{inside}}(H\lambda)\rangle - \langle T_{33}^{\text{outside}}(H\lambda)\rangle \cdot \langle T_{33}^{\text{inside}}\rangle$ is acoustic radiation stress in the polymer while $\langle T_{33}^{\text{outside}}\rangle$ is acoustic radiation stress in the surrounding medium. Accordingly, the stress versus strain relation can be rewritten as

$$\frac{f}{HL\sqrt{\lambda}} + (t_1 - t_3) = K(-\ln\lambda)^N. \tag{4}$$

This nonlinear equation actually contains the combined effects of material hardening and geometric nonlinearity on polymer deformation. Material hardening (i.e., monotonic increase of stress with increasing strain) is modeled by relation $K(-\ln\lambda)^N$. By contrast, geometric nonlinearity is reflected by $1/\sqrt{\lambda}$ and the implicit inclusion of $t_1(\lambda)$ and $t_3(\lambda)$.

To proceed further, the normalized mechanical force $F(= f/(KHL))$ and normalized acoustic force $Y(=p_0^2/\left(K\rho_0 c_0^2\right))$ can be expressed as functions of stretch λ, as

$$F(\lambda) = \sqrt{\lambda}(-\ln\lambda)^N - Y\sqrt{\lambda}(t_1 - t_3)\left(\frac{p_0^2}{\rho_0 c_0^2}\right)^{-1}, \tag{5}$$

$$Y(\lambda) = \frac{1}{t_1 - t_3}\frac{p_0^2}{\rho_0 c_0^2}\left[(-\ln\lambda)^N - F\frac{1}{\sqrt{\lambda}}\right]. \tag{6}$$

Eq. (5) specifies the dependence of F on λ at prescribed acoustic input while Eq. (6) specifies the dependence of Y on λ at prescribed mechanical force.

Variations of F and Y with λ are presented separately in Fig. 3(a) and (b), the former at fixed acoustic force and the latter at fixed mechanical force. Overall, the variation trends of these force–stretch curves stem from the competition between material hardening and geometric nonlinearity. In small deformation regime ($\lambda \approx 1$), material hardening dominates, causing both curves to increase monotonically. In large deformation regime ($\lambda \ll 1$), geometric nonlinearity dominates, causing the curves to decease, during which the softening effect offsets the material hardening effect. Competition of the two counteracting effects induces a peak on each force–stretch curve, thus giving rise to the critical condition for acoustomechanical instability to occur. Peaks in Fig. 3(a) correspond to necking instabilities caused by mechanical tensile force at fixed acoustic forces, whereas peaks in Fig. 3(b) correspond to pull-in instabilities caused by acoustic compressive force at fixed mechanical forces. As the prescribed acoustic force is increased, the critical mechanical force deceases but the critical stretch increases (Fig. 3(a)). In contrast, both the critical acoustic force and critical stretch decrease as the prescribed mechanical force is increased (Fig. 3(b)).

As acoustomechanical instability occurs when the incremental stiffness turns from positive to negative, we can determine the critical condition by setting $\partial F(\lambda)/\partial\lambda = 0$ at fixed Y or $\partial Y(\lambda)/\partial\lambda = 0$ at fixed F. For the problem of Fig. 1, the critical condition of mechanical force–stretch relation is

$$Y_c = \frac{\frac{1}{2}(-\ln\lambda_c)^N - N(-\ln\lambda_c)^{N-1}}{\frac{1}{2}(t_1 - t_3) + \lambda_c\frac{\partial(t_1 - t_3)}{\partial\lambda_c}}\frac{p_0^2}{\rho_0 c_0^2} \tag{7}$$

and the critical condition of acoustic force–stretch relation is

$$F_c = \frac{\frac{\partial(t_1 - t_3)}{\partial\lambda}(-\ln\lambda_c)^N\lambda_c + (t_1 - t_3)N(-\ln\lambda_c)^{N-1}}{\frac{\partial(t_1 - t_3)}{\partial\lambda}\lambda_c^{\frac{1}{2}} + \frac{1}{2}(t_1 - t_3)\lambda_c^{-\frac{1}{2}}}. \tag{8}$$

Eqs. (7) and (8) generalize the critical condition of necking instability induced by tensile mechanical force and the critical