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Optimal loading and protection of multi-state systems considering performance sharing mechanism

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ABSTRACT

Engineering systems are designed to carry the load. The performance of the system largely depends on how much load it carries. On the other hand, the failure rate of the system is strongly affected by its load. Besides internal failures, such as fatigue and aging process, systems may also fail due to external impacts such as nature disasters and terrorism. In this paper, we integrate the effect of loading and protection of external impacts on multi-state systems with performance sharing mechanism. The objective of this research is to determine how to balance the load and protection on system elements. An availability evaluation algorithm of the proposed system is suggested and the corresponding optimization problem is solved utilizing genetic algorithms.

1. Introduction

A multi-state series-parallel system consists of N sub-systems connected in series and each sub-system contains several multi-state elements connected in parallel [1–6]. The system fails if and only if at least one of the sub-systems cannot meet its required demand. In the literature, most of the works assume that each sub-system satisfies its own demand individually. However, many examples in real situations such as power systems, communication systems and data processing systems indicate that the surplus performance from one sub-system can be transmitted to other sub-systems that are experiencing the performance deficiency. This type of performance sharing mechanism was first studied in [7], and extended in [8,9].

Many research works have considered the element allocation and maintenance of the multi-state series-parallel system [5,10,11], but very limited work considered the effect of loading on the system elements. However, a majority of the engineering systems are designed to carry the load, such as coal conveyors, cargo trucks, and power generating units. The performance of such systems depends on the amount of load it is carrying. Besides, many studies have empirically shown that the failure rate of the system element is strongly affected by its working load [12,13]. Therefore, it is important to consider the effect of loading when analyzing the availability of multi-state systems.As a result, several recent research works have studied the optimal loading of multi-state systems [14–19].

Besides the internal failures such as aging process and fatigue, the system element may also fail due to external impacts such as natural disasters and terrorism [20]. One approach for improving the survivability of system elements is to make defensive investment to protect the elements [21–23]. The probability that an element is destroyed by the external impact is usually modeled as a function of the external impacts intensity and the protection effort allocated on the element [24–27]. The survival probability of an element is higher if more protection effort is allocated onto it. The optimal trade-off between investment into the maintenance and protection of the elements in a simple parallel system that is subject to both internal failure and natural disasters was studied in [27].

In this paper, we consider a multi-state series-parallel system with common bus performance sharing as shown in Fig. 1. The performance of each sub-system is the cumulative performance of the elements within the sub-system. If the performance of any sub-system is more than its demand, the surplus performance can be transmitted to any other deficient sub-system via the common bus. However, the total amount of the performance that can be transmitted among different sub-systems is subjected to the capacity of the common bus. In this research, the capacity is also assumed to be a random variable since the common bus may be

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Nomenclature

e_i :	multi-state element i
E_j :	the set of multi-state elements located at sub-system j
L_i :	the load on e_i
$g(L_i)$:	the performance of e_i when its load is L_i
$L_{i,\min}$:	the minimum allowable load on e_i
$L_{i,\max}$:	the maximum allowable load on e_i
N :	the number of sub-systems connected in series
M :	the total number of multi-state elements
G_i :	the random performance of e_i
O_j :	the random performance of sub-system j
W_j :	the random demand of sub-system j
C :	the random transmission capacity of the common bus performance sharing system
S_j :	the random performance surplus of sub-system j
D_j :	the random performance deficiency of sub-system j
Z :	the random amount of redistributed performance
$\hat{D}$:	the system deficiency after redistribution
x_i :	the protection effort allocated to e_i
f :	constant frequency of external impacts
d :	expected impacts intensity
c_{ri} :	constant repair cost of e_i for internal failures
c_{p_2i} :	constant repair cost of e_i for failure caused by external impacts
c_{p_1i} :	unit protection cost of e_i
t_{ri} :	constant repair time of e_i for internal failures
t_{pi} :	constant repair time of e_i for failure caused by external impacts
T :	system lifetime
C_T :	the total system cost
A^* :	the pre-specified system availability requirement
$\theta(j)$:	the number of states of the demand of sub-system j
q_{jr} :	the probability that the demand of sub-system j is in state r
w_{jr}:	the demand of sub-system j when it is in state r
Q :	The number of states of the common bus performance sharing system
α_β:	the probability that the common bus performance sharing system is in state β
ς_β:	the transmission capacity when the common bus performance sharing system in state β

concurrently used by several systems.The performance of any system element depends on the load it is carrying. In general, the working performance of each element increases with increasing load. However, increasing the load will also increase the failure rate of the element, and thus reduce the availability of the element. Therefore, the expected performance of the element is not a monotonic function of its load. Besides internal failures, the system elements are also subject to external impacts that occur with a constant frequency. Hence, it is important to consider providing protection to the system elements to increase their survivability against external impacts. For example, if the external impacts are from earthquakes, anti-seismic devices can be installed to protect system elements.

Comparing with other works on common bus systems, we have considered both the effects of internal failure and external impacts on the system elements. To increase the system availability, the system elements are protected and immediately repaired in case of failure. Besides, the effect of load on element failure rate is considered. Since increasing load can increase the element failure rate, but also increases the element performance, the relationship between load and the system availability is already non-monotonic. With the incorporation of the external impact, it is impossible to give an intuitive answer to how to load and protect the system element. Therefore, it is essential to propose a framework to solve the joint optimization problem of system loading and protection.

The rest of the paper is organized as follows. Section 2 discusses the load dependent failure rate and performance. Section 3 provides the availability modeling of the multi-state series-parallel system with performance sharing. Section 4 formulates the joint optimization model of loading and protection. The availability evaluation algorithm of the proposed system based on universal generating function is suggested in Section 5. Section 6 proposes the optimization technique. Numerical experiments are provided in Section 7. The paper is concluded in Section 8.

2. Load dependent failure rate and performance

Every multi-state element e_i considered in this paper is able to carry load of different values. However, element e_i can only be in two states: failure state with zero performance and functioning state with working performance $g_i(L_i)$, where L_i is the load carried by element e_i and $g_i(L_i)$ which denotes the performance of element e_i is a function of load L_i. The load on element e_i can vary from $L_{i,\min}$ to $L_{i,\max}$, where $L_{i,\min}$ and $L_{i,\max}$ denote the minimum and maximum allowable load on element e_i respectively. This is the reason why element e_i is considered to be multi-state. As discussed earlier, the expected performance of element e_i is not a monotonic function of the load L_i.

It can be difficult to distinguish the load and the performance in some situations since they can be considered the same. For example, the pressure on a pipe which carries fluid in a laminar flow mode is proportional to the volume of the flow, i.e., the throughput (performance) is proportional to the pressure (load). However, the load-performance relationship can be nonlinear. For instance, consider a pipe carrying fluid in a turbulent flow mode. The pressure on the pipe is a non-linear function of volume of the flow.

The load dependent performance function $g_i(L_i)$ can take different expressions depending on the situations. Without loss of generality, the function measuring relationship between the performance and the load can be assumed as follows.

$$g_i(L_i) = a_i + c_i L_i \tag{1}$$

where a_i and c_i are the coefficients of the linear equation.

To analyze the availability of systems made up by elements with load dependent failure rates, the load failure relationship must be known beforehand since the availability of each element can be derived based on its failure rate. In the literature, the accelerated life test models play an important role in determining the load-failure rate relationships. The existing accelerated life test models are summarized in [28]. In this paper, the commonly used proportional hazard model (PHM) will be discussed and used in the numerical experiments.

PHM, which was first proposed in [29], has received popularity in the field of reliability engineering in recent years [30–32].PHM states that the failure rate of an element is the product of the baseline hazard rate and factors based on the conditions. In general, PHM can be expressed as follows.

$$h(t|\boldsymbol{X}) = h_0(t) \cdot e^{(\beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_d X_d)} \tag{2}$$

where $h_0(t)$ denotes the baseline hazard rate as a function of time. $X_1, \cdots, X_d$ are the factors that affect the hazard rate function and $\beta_1, \cdots, \beta_d$ are the corresponding coefficients.