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Letter

On the stress–strain states of cellular materials under high loading rates

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HIGHLIGHTS

- Dynamic impact of cellular materials is analyzed with wave propagation technique.
- Time history of particle velocity and stress–strain history curves are obtained.
- The dynamic stress–strain states obtained verify the validity of the dynamic-rigid-plastic hardening (D-R-PH) model.

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ABSTRACT

A virtual Taylor impact of cellular materials is analyzed with a wave propagation technique, i.e. the Lagrangian analysis method, of which the main advantage is that no pre-assumed constitutive relationship is required. Time histories of particle velocity, local strain, and stress profiles are calculated to present the local stress–strain history curves, from which the dynamic stress–strain states are obtained. The present results reveal that the dynamic-rigid-plastic hardening (D-R-PH) material model introduced in a previous study of our group is in good agreement with the dynamic stress–strain states under high loading rates obtained by the Lagrangian analysis method. It directly reflects the effectiveness and feasibility of the D-R-PH material model for the cellular materials under high loading rates.

Cellular materials have been extensively used in crashworthiness application due to their considerable capacity of energy absorption [1–4]. The stress–strain relation of cellular materials under low-velocity compression shows three distinct stages of deformation, namely elastic, long plateau, and densification stages. The dynamic behavior of cellular materials under high-velocity loading is dominated by inertia effect, which leads to the deformation localization and stress enhancement [5]. The dynamic features can be well explained by the 1D shock wave models [5–8], in which the rigid-perfectly plastic-locking (R-PP-L) shock wave model involving only two material parameters (locking strain ε_L and plateau stress σ_{pl}) is the most popular one. However, the R-PP-L shock wave model can only characterize the compressive behavior of cellular materials in a first-order approximation [9,10]. Recently, Zheng et al. [11] proposed a rigid-plastic hardening (R-PH) material model (an R-PH idealization) and a dynamic-rigid-plastic hardening (D-R-PH) material model (a D-R-PH idealization) to describe the stress–strain behaviors of cellular materials under low-velocity compression and under high-velocity impact, respectively, and revealed different deformation mechanisms to explain the loading-rate effect of cellular materials. The stress–strain relation of the D-R-PH idealization [11] is written as

$$\sigma = \sigma_0^{\mathrm{d}} + \frac{D\varepsilon}{(1-\varepsilon)^2}, \tag{1}$$

where σ_0^{d} is the dynamic initial crushing stress and D the strain hardening parameter. They found that the stress–strain states of cellular materials are essentially loading-rate dependent [11]. In this study, we aim to present further evidence on the loading-rate effect of cellular materials.

In the shock wave models for cellular materials [5–11], the shock wave assumption is an idealized assumption, which is appropriate only for the case with a relatively high impact velocity. The wave propagation technique may provide an opportunity to investigate the strain–stress behavior of cellular materials without any pre-assumptions. The Lagrangian analysis method [12–14] gets the favor of most researchers because no constitutive relation is required. However, the boundary/initial conditions are

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required for the traditional Lagrangian analysis method as it involves integral operations. To overcome this difficulty, a method combining the Hopkinson pressure bar technique and Lagrangian analysis was developed by Wang et al. [15], from which the physical quantities (particle velocity, stress, etc.) at the boundary of the specimen can be obtained simultaneously. This wave propagation technique, namely the "$1sv+nv$" Lagrangian method, contains a dual-information of stress and velocity at one position often located at the boundary of specimen and n particle velocity profiles. Due to the big difference in wave impedance between the elastic bar and cellular materials, another experimental technique (Taylor–Hopkinson bar with knowing initial conditions) [16] was carried out to investigate the dynamic response of aluminum foam. Unfortunately, the experimental data of cellular materials under high impact velocity, say $v > 200$ m/s, are hardly obtained and the other limitations like the digital image correlation accuracy also restrict its application. To make up the deficiencies in experimental study, the mesoscopic finite element (FE) model is employed in this paper and the Lagrangian analysis is applied to study the stress–strain behavior of cellular materials under high-velocity impact.

In 1D stress wave propagation theory [17], the mass and momentum conservation conditions in Lagrangian coordinate are expressed as

$$\frac{\partial v}{\partial X} = -\frac{\partial \varepsilon}{\partial t} \tag{2}$$

and

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial \sigma}{\partial X}, \tag{3}$$

respectively, where v, ε, and σ are particle velocity, strain, and stress, respectively; X is Lagrangian coordinate; t is time; ρ_0 is the initial density of cellular specimen. Here, the stress and strain are defined as positive for compressive case, and negative for tensile case. Eqs. (2) and (3) are the basic equations of the Lagrangian analysis method, from which the stress and strain relation can be built with the aid of particle velocity field. Since the variables are connected by their first-order partial derivatives, the integral operations are requisite and the boundary/initial conditions are required to determine the integral constants. Supposing detailed information of particle velocity $v(X_i, t_j)$ at Lagrangian coordinates X_i $(i = 1, 2, \ldots, n)$ and time t_j $(j = 1, 2, \ldots)$ has been measured, and then the strain and stress fields can be determined from Eqs. (2) and (3), written in the difference form

$$\varepsilon_{i,j+1} = \varepsilon_{i,j} - \frac{\Delta t}{2\Delta X}\left(v_{i+1,j} - v_{i-1,j}\right), \tag{4}$$

and

$$\sigma_{i+1,j} = \sigma_{i,j} - \frac{\rho_0 \Delta X}{2\Delta t}\left(v_{i,j+1} - v_{i,j-1}\right), \tag{5}$$

where ΔX is the grid size and Δt the time step. It is worth noticing that the partial derivatives ($\partial\sigma/\partial X$ and $\partial v/\partial X$) may be inaccurate when the distance of two adjacent Lagrangian positions is not small enough, as for most experimental cases. Grady [14] introduced the path-line method to improve the computational accuracy of Lagrangian analysis, in which the first order derivatives containing variable X is changed to the partial derivatives containing variable t by the total differentiation along the path-line. If there is sufficient data obtained from a test, the path-line method is not necessary. The FE simulations can offer a detailed particle velocity field and the boundary stress.

The mesoscopic FE model [11] is employed to perform the quasi-static compression and dynamic impact of cellular materials. Closed-cell foam models are generated by employing the 3D Voronoi technique, see Ref. [11] for details. The cell-wall material of the Voronoi structure is assumed to be elastic, perfectly plastic with density $\rho_s = 2770$ kg/m^3, Young's modulus $E = 69$ GPa, Poisson's ratio $\nu = 0.3$, and yield stress $\sigma_{ys} = 170$ MPa. The relative density of the Voronoi structure used is $\rho_0/\rho_s = 0.1$ and cell walls have a uniform thickness. The cellular specimen used in this study is constructed in a volume of $30 \times 20 \times 20$ mm^3 with 600 nuclei, and the average cell size $d \approx 3.34$ mm. The ABAQUS/Explicit software is used to perform the numerical simulations.

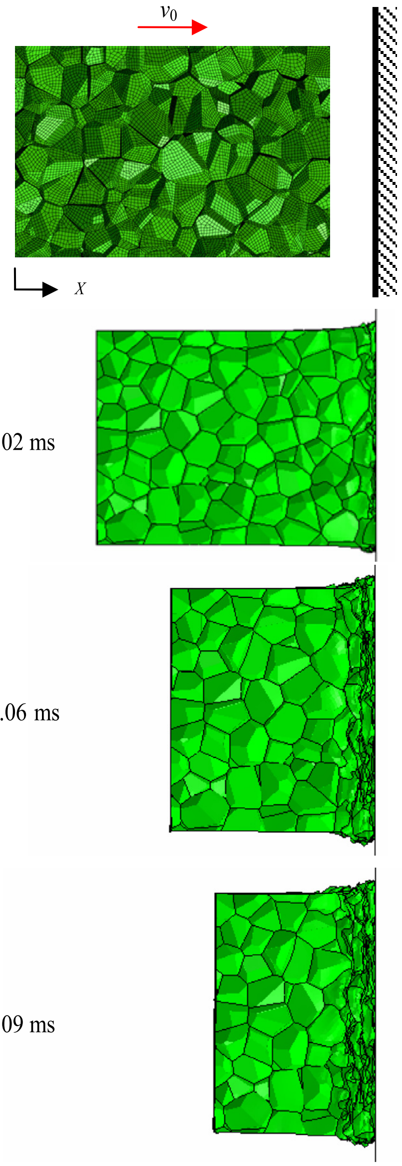

Fig. 1. The Taylor impact test scenario and deformation patterns.

A Taylor impact test scenario of cellular materials is performed, as schematically represented in Fig. 1. In the virtual test, the specimen travels at an initial velocity $v_0 = 250$ m/s and impacts a fixed rigid target. Some deformation patterns are presented in Fig. 1. The velocity profiles at all element nodes can be extracted from the FE simulations, and the stress, strain, and velocity profiles at free end can also be acquired simultaneously. Supposing the X coordinate is established at the free end and the area of specimen is considered to be unchanged throughout the test, then the dynamic strain–stress states can be investigated by using the "$1sv + nv$" Lagrangian analysis method.