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Letter

Performance of global look-up table strategy in digital image correlation with cubic B-spline interpolation and bicubic interpolation

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HIGHLIGHTS

- Global look-up table strategy is used to accelerate B-spline interpolation in digital image correlation (DIC).
- Performance of the strategy is evaluated theoretically and experimentally.
- The strategy is found a superior substitute for the one with bicubic interpolation.

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ABSTRACT

Global look-up table strategy proposed recently has been proven to be an efficient method to accelerate the interpolation, which is the most time-consuming part in the iterative sub-pixel digital image correlation (DIC) algorithms. In this paper, a global look-up table strategy with cubic B-spline interpolation is developed for the DIC method based on the inverse compositional Gauss–Newton (IC-GN) algorithm. The performance of this strategy, including accuracy, precision, and computation efficiency, is evaluated through a theoretical and experimental study, using the one with widely employed bicubic interpolation as a benchmark. The global look-up table strategy with cubic B-spline interpolation improves significantly the accuracy of the IC-GN algorithm-based DIC method compared with the one using the bicubic interpolation, at a trivial price of computation efficiency.

Digital image correlation (DIC) is a non-contact and full-field optical measurement technique which has found a wide variety of applications [1–5]. To achieve high accuracy and precision of measurement, various sub-pixel DIC algorithms have been developed in the past years, among which the one based on the inverse compositional Gauss–Newton (IC-GN) algorithm [6] has become a popular DIC algorithm nowadays due to its superior performance (accuracy, precision, and computation efficiency) [1,7–9]. As an iterative optimization algorithm, the performance IC-GN algorithm depends heavily on the interpolation, whereby the intensity map of warped target subset in the deformed image is reconstructed at sub-pixel locations during each iteration step: (i) the bias of interpolation affects directly the accuracy of the sub-pixel DIC algorithms. Researchers studied the influence of various interpolation algorithms on the accuracy of the sub-pixel DIC methods quantitatively [10–14]. Their work demonstrates that the cubic B-spline interpolation leads to considerably less bias of the obtained results compared with the bicubic interpolation; (ii) the interpolation is the most time-consuming part of the iterative procedure. Recently, Pan and Li [15] proposed a global look-up table strategy to accelerate the bicubic interpolation for the iterative sub-pixel DIC algorithm. By using a pre-computed table of interpolation coefficients, the interpolation times for processing a pair of subsets can be reduced for almost two orders of magnitude. The similar strategy can be also employed to alleviate the influence of the non-linear error in processing of digital fringe patterns [16].

In this paper, a global look-up table strategy with cubic B-spline interpolation is developed for the IC-GN algorithm-based DIC method. The developed DIC method is compared with the one using the same strategy but with bicubic interpolation, at the aspects of accuracy, precision and efficiency.

In both cubic B-spline interpolation and bicubic interpolation, a piece of smooth surface of intensity is constructed over each 2×2-pixel grid ($q_{ij}q_{(i+1)j}q_{i(j+1)}q_{(i+1)(j+1)}$) in the image, as illustrated in Fig. 1. The surface guarantees that it goes through the four

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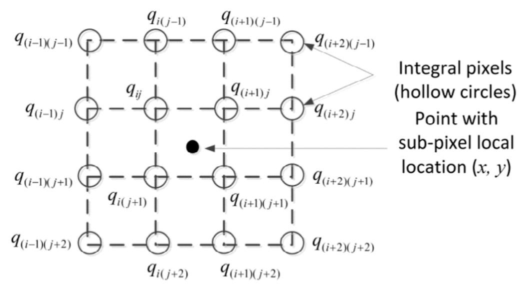

Fig. 1. Illustration of 4×4-pixel grid for calculation of 16 interpolation coefficients.

integer-pixel corners and its first and second partial derivatives are continuous. The intensity $t(x, y)$ at sub-pixel location on this surface can be expressed as a polynomial form

$$t(x,y)=\sum_{m=0}^{3}\sum_{n=0}^{3}a_{mn}x^m y^n, \tag{1}$$

where (x, y) denotes the local coordinates of the sub-pixel location with respect to the upper-left corner of a 2×2-pixel grid (Fig. 1). $\boldsymbol{a} = \left[a_{ij}\right]$ is a matrix containing 16 interpolation coefficients calculated using the intensity information $\left[q_{ij}\right]$ of a grid of 4×4 integer pixels surrounding this sub-pixel location.

For the cubic B-spline interpolation, the coefficient matrix $\boldsymbol{a}$ of each grid can be calculated according to Ref. [17]

$$\boldsymbol{p}=\boldsymbol{BCQC'B'}, \tag{2}$$

where

$$\boldsymbol{B}=\frac{1}{6}\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 0 & 3 & 0\\ 1 & 4 & 1 & 0\end{bmatrix},$$

$$\boldsymbol{C}=\frac{1}{56}\begin{bmatrix}71 & -19 & 5 & -1\\ -19 & 95 & -25 & 5\\ 5 & -25 & 95 & -19\\ -1 & 5 & -19 & 71\end{bmatrix},$$

$$\boldsymbol{Q}=\begin{bmatrix}q_{(i-1)(j-1)} & q_{(i-1)j} & q_{(i-1)(j+1)} & q_{(i-1)(j+2)}\\ q_{i(j-1)} & q_{ij} & q_{i(j+1)} & q_{i(j+2)}\\ q_{(i+1)(j-1)} & q_{(i+1)j} & q_{(i+1)(j+1)} & q_{(i+1)(j+2)}\\ q_{(i+2)(j-1)} & q_{(i+2)j} & q_{(i+2)(j+1)} & q_{(i+2)(j+2)}\end{bmatrix}.$$

Matrix $\boldsymbol{a}$ has a simple relation with $\boldsymbol{p}$, i.e.

$$a_{ij}=p_{(3-i)(3-j)}. \tag{3}$$

For the bicubic interpolation, the construction of coefficient matrix $\boldsymbol{a}$ uses not only the intensity at neighboring integer-pixels but also their gradients. A detailed description can be found in Ref. [18].

A global look-up table consisting of $(M-1)\times(N-1)$ elements can be pre-computed on a $M \times N$-pixel speckle image. By repeatedly referring to this table during the iteration, considerable redundant calculations of interpolation coefficients are avoided. Obviously, this is a trade-off between the computation efficiency and memory usage. However, for a 768×576-pixel speckle image the global look-up table requires additional memory of about 56 MB, which could be a trivial expense to current computers.

Table 1 lists the number of operations required to pre-compute matrix $\boldsymbol{a}$ of a 2×2-pixel grid for the two interpolation algorithms. It can be found that the operations for the bicubic interpolation are about 50% less than those for the cubic B-spline interpolation. Moreover, the pre-computation of matrix $\boldsymbol{a}$ for the cubic B-spline interpolation needs an additional transform from matrix $\boldsymbol{p}$, which leads to extra operations.

The bias raised in the two interpolation algorithms can be compared using the model proposed by Su et al. [14], in which

Table 1
Number of operations in cubic B-spline and bicubic interpolation.

Interpolation algorithm	Operations of addition and subtraction	Operations of multiplication and division
Bicubic	104	108
Cubic B-spline	153	168
Relative difference	47%	55%

an interpolation bias kernel $E_{\mathrm{ib}}(v_x, v_y)$ is used as an indicator. It is defined as follows, neglecting the aliasing effect along the y-axis

$$E_{\mathrm{ib}}(v_x,v_y)=(v_x-1)\hat{\varphi}(v_x-1,v_y)-(v_x+1)\hat{\varphi}(v_x+1,v_y)$$
$$+\hat{\varphi}(v_x,v_y)\hat{\varphi}(v_x-1,v_y)+\hat{\varphi}(v_x,v_y)\hat{\varphi}(v_x+1,v_y), \tag{4}$$

where v_x and v_y are the frequency of the signals along x-axis and y-axis. Since the lowest period in a digital image is two pixels, the domain of v_x and v_y is limited in $(-0.5,\ 0.5)$. $\hat{\varphi}(v_x, v_y)$ represents the interpolation transfer function, which is the Fourier transform of the convolution kernel of an interpolation function. The interpolation transfer functions of the two interpolation algorithms can be expressed as equations in Box I.

Figure 2(a) and (b) shows the surfaces of $E_{\mathrm{ib}}(v_x, v_y)$ for the cubic B-spline interpolation and the bicubic interpolation. A clearer comparison between the value of the two interpolation bias kernels is performed on the section of $v_y = 0.1$ and $v_y = 0.25$, as shown in Fig. 2(c) and (d). It can be seen that the value of cubic B-spline interpolation bias kernel is lower than that of bicubic interpolation bias kernel in the low-frequency region ($-0.36 < v_x < 0.36$). As we know that the energy of an image usually concentrates in the low-frequency region, thus the cubic B-spline interpolation algorithm can reach smaller bias than the bicubic interpolation algorithm for most of speckle images.

Figure 3 shows a practical speckle image with a size of 768×576 pixels. Using Fig. 3 as the reference image, twenty target images were generated by translating it in Fourier domain according to the shift theorem [10], with pre-set sub-pixel displacements along x-axis from 0 to 1 pixel. The displacement between every two successive images was set to be 0.05 pixels.

The IC-GN algorithm-based DIC method with the two interpolation algorithms was programmed using C++ language and run on a desktop computer equipped with AMD FX-4300 CPU (4 cores, 3.8 GHz) and 8.0 GB RAM. The initial guess for the IC-GN algorithm is estimated using the Fourier transform-based cross correlation (FFT-CC) algorithm. Details of this DIC method can be found in Ref. [9].

The bias caused by the two interpolation algorithms is evaluated using the mean bias error of the calculated results, defined as:

$$e_u=\frac{1}{M}\sum_{i=1}^{M}\left(u_i-u_{\mathrm{d}}\right), \tag{6}$$

where M denotes the number of points of interest (POIs, center of a subset). In this work, a 33×33-pixel subset is employed in the DIC computation and 15264 POIs are set in each speckle image. u_i is the calculated displacement at the ith POI, and u_{d} is the pre-set displacement.

Figure 4(a) shows the mean bias errors of the calculated displacements. The dependence of mean bias error on the sub-pixel displacement is in the form of a sinusoidal function, which stems from the interpolation bias, as explained in Refs. [10,14]. Distinct gap can be observed between the two curves of mean bias errors. The magnitude of mean bias errors caused by the cubic B-spline interpolation can be up to 46% lower than those by the bicubic interpolation, indicating a markedly higher accuracy. The gap also shows dependence on sub-pixel displacement. The