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A variational hydraulic fracturing model coupled to a reservoir simulator

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ABSTRACT

A variational fracture model coupled to an external reservoir simulator through variable exchange is presented. While convergence is not optimal without Jacobian matrices with which fully coupling can provide, the presented coupling scheme is versatile enough that the reservoir simulator could be easily replaced with any other simulator. A variational approach to fracture is introduced first by comparison to the classic Griffith criteria, and is then expanded to include poro-elasticity and in-situ stresses that are required in hydraulic applications. The coupled code has been tested against existing analytical solutions of fluid-driven fracture propagation. Finally, illustrative examples are shown to demonstrate that the methodology's ability to simulate multi fracture interaction with the unified approach for turning and merging fractures.

1. Introduction

In predicting fracture propagation during well stimulation or water/waste injection, it is crucial to properly understand the behavior of the fractures induced. For example, the assumption of a single planar fracture propagation often leads to an estimation of an unrealistically long fracture given the volume of injection in water injection operation, which then leads to an over-specification of water treatment program with higher capital and operating expenditure,[1,2] or not accounting for interaction with existing fractures (either man-made or pre-existing) may result in unfavorable well spacing for tight rock development, which can lead to increased number of wells or hydraulic stimulation stages.[3,4,5] Thus the urgency to develop predictive capabilities for complex hydraulic fracture propagation (turning, merging, and branching) is increasing in the industry as well as the requirement for complex flow behaviors such as fluid phase change or particle deposition in porous media.

To date, most hydraulic fracturing simulations have focused upon the problem of a single mode-I fracture on a vertical plane driven by a pressurized fluid applying Linear Elastic Fracture Mechanics (LEFM) as propagation criteria coupled with Poiseuille's equation of fluid flow in the fracture and Carter's equation for leak-off to the formation. A thorough historical background and review of the LEFM based approach on 2D, pseudo 3D, and full 3D fracture modeling has been conducted by Adachi.[6] Additionally models that consider flow in both reservoir and fracture flow instead of treating fluid leak off with Carter's equation have been proposed.[7,8,9,10] Lujun et al.[7] applied hydraulic force as boundary force on the fracture placed in the boundary. A cohesive element approach on a planar fracture[8] and a turning mix-mode fracture, and sub-grid enrichment of finite element method[10] have been also studied.

For hydraulic fracture models in the presence of natural pre-existing fractures, Kresse et al.[11] utilized pseudo 3D approach for the propagating main fracture and semi-analytical crossing criteria for interaction with pre-existing natural fractures. Natural fractures were treated as closed weak planes and its mechanical interaction with hydraulic fractures was computed with a 2D Boundary Element Method (BEM) by incorporating empirically derived 3D effects. McClure et al.[12] applied the LEFM approach for the criteria for hydraulic fracture initiation and propagation on the prescribed plane using fixed grid for growth of fracture and utilized BEM for stress disturbance by natural fracture opening and shearing. Similarly to Kresse et al. [11], semi-analytical crossing criteria proposed by Gu and Weng[13] were used for interaction between hydraulic fractures and natural fractures. A Discrete Element Method (DEM) has been also applied to hydraulic fracturing with natural fractures.[14] In the DEM framework, hydraulic fractures propagate along prescribed element boundaries when a stress intensity factor meets the criteria, and the natural fracture opening is estimated using a Coulomb friction model. While the stress shadow effects of opening fractures were accounted in these studies, poroelastic impacts induced by leak-off were not included.

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Recently, simulation approaches to complex fracture(s) along unknown path(s) have been developed using different techniques such as a BEM or an extended finite element method (XFEM). Wu and Olson[15] modeled 2D fractures that propagate in both mode-I and II using 3D correction in the BEM formulation. Complex single fracture propagation in 3D has been developed by Rungamornrat and Mear.[16] Their model has been extended to multiple fracture propagation and interaction by Castonguay et al.[17] An XFEM has been first applied to hydraulic fracture by incorporating pressure forces along a line fracture in 2D impermeable media by Lecampion[18] for stationary fracture. Dahi-Taleghani and Olson[19] implemented the XFEM for propagating fracture in 2D and Gordeliy and Peirce[20] extended the methodology to include solid-fluid interaction at the fracture tip. While these methods are appealing for not requiring a priori knowledge of the crack path, the BEM imposes restrictions on heterogeneity in material properties and handling of merging fractures remains as a perplexing challenge.

The variational approach to fracture was originally proposed by Francfort and Marigo[21] and was numerically implemented by Bourdin et al.[22,23] using a "phase-field" approach. This approach is capable of tracking arbitrary number of fractures in any geometry, regardless of the propagation mode. It was extended to hydraulic fracturing in impermeable media by accounting for the work of the pressure forces applied along the fracture in Bourdin et al.,[24] where it was shown that explicit properties such as fracture aperture or critical propagation pressure could be retrieved from the phase field. Phase-field's implicit representation of the fracture system has proved useful in hydraulic fracturing simulations and its initial application has been followed by many others.[25,26,27,28,29,30] Wheeler et al.[25] extended the phase-field model to porous media by including poroelastic terms in the total energy. Its implementation has been enhanced with adaptive finite element in 2D[27] and 3D.[29] Wick et al.[30] coupled the methodology to a reservoir simulator using an indicator function for fractures. Miehe et al.[28] coupled porous media flow with the phase-field hydraulic fracture using permeability decomposition and the unilateral contact condition.[31] Mikelic et al.[26] fully coupled the three-field problems and modified the total energy functional from their previous studies.

In this article, we show how a phase-field fracture model of fracture and an existing reservoir simulator can be coupled with minimal modifications. The main motivation for this approach, over that seeking to leverage the phase-field description of the fracture in the flow model,[32,33] is that it allows reusing a feature-rich, validated reservoir simulator. The proposed coupling is iterative, and does not allow sharing information on the Jacobians. However, it uses the same computational grid for the mechanics and flow solvers, so that constructing an explicit mesh of the fracture geometry is not required, and is reasonably efficient. In the sequel, we describe the construction of the phase field model, the implementation of the coupled simulator, and illustrate the ability of the this approach to handle critical features such as crack propagation along unknown path in two and three dimension, ability to handle interactions between stimulated and pre-existing fractures, and nucleation of new add-cracks.

2. Mathematical model

2.1. Variational approach to fracture

Consider a domain Ω in 2 or 3D, occupied by a brittle linearly elastic material with stiffness tensor $\mathbf{C}$, and a fracture set Γ and critical energy release rate (fracture toughness) G_c (Fig. 1a). Let $\mathbf{f}(t,x)$ denote a time-dependent body force applied to Ω, $\boldsymbol{\tau}(t,x)$ a surface force applied to a part $\partial_N\Omega$ of its boundary whose normal vector is $\mathbf{n}_\Omega$, and $\mathbf{g}(t,x)$ a prescribed boundary displacement on the remaining part $\partial_D\Omega$.

The stress-strain relationship is given as

$$\boldsymbol{\sigma} = \mathbf{C}e(\mathbf{u}) \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress field, and $e(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla^T\mathbf{u})$ is the strain field, and the equilibrium equations in strong form are:

$$-\nabla\cdot\boldsymbol{\sigma} = \mathbf{f} \text{ in } \Omega/\Gamma, \tag{2}$$

$$\boldsymbol{\sigma}\cdot\mathbf{n}_\Omega = \boldsymbol{\tau} \text{ on } \partial_N\Omega, \tag{3}$$

$$\mathbf{u} = \mathbf{g} \text{ on } \partial_D\Omega. \tag{4}$$

No stress and positive or zero displacement discontinuity on the fracture surface are assumed:

$$\boldsymbol{\sigma}\cdot\mathbf{n} = 0 \text{ on } \Gamma, \tag{5}$$

$$(\mathbf{u}^+ - \mathbf{u}^-)\cdot\mathbf{n} \ge 0 \text{ on } \Gamma. \tag{6}$$

where u^+ and u^- are the displacement on each surface of the fracture. The total external work W consists of the work by the body force and the external load and is defined for any kinematically admissible displacement $\mathbf{u}$ as

$$W(\mathbf{u}) := \int_\Omega \mathbf{f}\cdot\mathbf{u}\,d\Omega + \int_{\partial_N\Omega} \boldsymbol{\tau}\cdot\mathbf{u}\,ds. \tag{7}$$

The potential energy E is given by the elastic energy of the system subtracting the external work as:

$$E(\mathbf{u},\Gamma) := \int_{\Omega/\Gamma} e(\mathbf{u}) : \mathbf{C}e(\mathbf{u})\,d\Omega - W(\mathbf{u}). \tag{8}$$

The variational approach to fracture proposed by Francfort and Marigo[21] defines the total energy as the sum of the potential energy and the surface energy required to create a fracture set Γ:

$$F(\mathbf{u},\Gamma) = E(\mathbf{u},\Gamma) + G_c H^{N-1}(\Gamma), \tag{9}$$

where H is the Hausdorff measure of Γ providing the fracture length in 2D ($N=2$) and the surface area in 3D ($N=3$).

In the Griffith theory, a single fracture in 2D with the length l is considered and the elastic energy release rate G can be calculated along an a priori known fracture path as:

$$G = \frac{dE}{dl}. \tag{10}$$

The criteria state that the fracture will propagate when $G = G_c$ and not when $G < G_c$, which is nothing but the criticality of the total energy of the system:

$$F(\mathbf{u},l) = E(\mathbf{u},\Gamma) + G_c l. \tag{11}$$

In the variational setting, the Griffith criteria are recast as the minimum of the total energy (Eq. (9)) with respect to any admissible displacement field $\mathbf{u}$ and any fracture set subject to an irreversibility condition. Namely, at any time step t_i, (u_i, Γ_i) is sought as the solution of the minimization problem:

$$\inf_{\begin{cases}\mathbf{u} \text{ kinematically admissible} \\ \Gamma_j \subset \Gamma \text{ for all } j < i\end{cases}} F(\mathbf{u},\Gamma) \tag{12}$$

It should be emphasized that in Eq. (12) no assumption on the geometry of the fracture is made a priori. Therefore, fractures are allowed to take an arbitrary path (turning or bifurcating), and the number of fracture does not need to remain constant, which