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Hydraulic fracture numerical model free of explicit tip tracking

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ABSTRACT

The paper addresses the problem of modeling a fracture propagation in linear elastic porous media driven by injection of non-Newtonian power-law fluid. The model involves the lubrication theory equation expressing the mass conservation of fluid and a nonlocal singular integral relation for the fracture aperture as a functional of the fluid pressure. We assume only the viscosity-dominated case in which the fracture toughness is neglected. This allows considering the fracture as an opened part of a preexisting closed fracture of larger length. The numerical method consists in solving equations of the model over the whole length of the preexisting fracture without distinguishing the tip region of the opened part. The problem is solved via the finite element method. The weak formulation of the solution allows pressure singularity with the required asymptotics near the fracture tip. Comparison of the numerical and the exact self-similar solutions in the case of a constant flow rate, zero fluid leakoff, and zero fracture toughness reveals the accuracy of the approximate solution to at least $O(h^{1/2})$, where h is the maximal linear size of the grid cells. As an illustration, we also demonstrate the numerical experiments with periodic fluid injection and variable fluid efficiency.

1. Introduction

Hydraulic fractures are common objects in the oil and gas industry and geomechanics. They are created and propagate due to the pressure of fluid injected either naturally (volcanoes), or artificially (hydraulic fracturing of oil- and gas-bearing formations). The main problem of the mathematical modeling of such fractures is the coupling between the fluid motion inside the fracture and the elastic interaction of the fluid pressure and rock stresses that determines the fracture opening. Industrial demands evoked broad interest in the problem of hydraulic fracture modeling, which is now expressed in numerous publications, both engineering and scientific (see [1,2]).

Among the most often used approaches are the 1D models of Perkins, Kern, and Nordgren (PKN) [3,4]; Khristianovich, Zheltov, Geertsma and de Klerk (KGD) [5,6]; and the generalization of the latter model to the radial geometry (penny-shaped fractures [2]). Although very well known and analyzed in many papers, these models still attract scientific interest to the correct and fast numerical implementation and possible generalizations to pseudo-3D cases, non-Newtonian fluid, presence of fluid leakoff through fracture walls, various fracture propagation criteria, etc. (see [2,7,8] and references therein). Another feature of the KGD models which is broadly discussed in the literature is the pressure singularity at the fracture tip and the asymptotic behavior of pressure and fracture aperture near the tip (see [9]).

In our paper, we focus on the KGD approach for the calculation of fracture propagation. Two obvious problems of this model are the pressure singularity at the tip and the coupling of the differential mass conservation equation with a nonlocal integral relation between the fluid pressure and the fracture aperture. The resulting nonlinear integro-differential operator is known to be noncontracting [10], which makes the most straightforward iterative procedure consisting of calculation of the fluid pressure for the fixed aperture at the first step, and the subsequent correction of the aperture for the known pressure at the second step, to be divergent. The incompressibility of fluid imposes volume balance conditions on the growing fracture length and width and the flow of fluid inside the fracture. The numerical algorithm requires thorough selection of the time step and of the step of the fracture enlargement to converge iterations to the balanced flow rates (see, for example, the numerical Loramec code [11]).

The goal of the present paper is to propose a numerical algorithm that accounts for all the problems mentioned above. We observe only the zero-toughness case [12], which implies that

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no energy is required to break the formation. This approximation becomes valid after a short stage of fracture initiation [13].

We assume the fracture to be an opened part of a preexisting closed fracture of larger length. The mass continuity equation and the elasticity relation are identically satisfied over the closed part. This allows us to solve the main integro-differential equation for pressure directly, over the whole length of the preexisting fracture without distinguishing the tip of the opened part. The weak formulation of the solution and the numerical algorithm make it possible to incorporate singular functions with the required asymptotic [9], i.e., the pressure singularity and proper aperture at the fracture tip arises here naturally in the solution of the problem. This differs our algorithm from the existing hydraulic fracture simulators [14].

We tested our algorithm on the exact self-similar solution [15] and showed good agreement and the convergence of the numerical solution. The largest error of the solution is observed near the fracture tip (two to three grid cells). It becomes significantly smaller over the remaining opened fracture length.

The algorithm is applicable to non-Newtonian power-law fluid, nonconstant inflow rate, and nonzero fluid leakoff. As an illustration, we demonstrate the simulation of fracture opening due to the periodic fluid injection in the presence of fluid leakoff with the prescribed fluid efficiency (the ratio of the current fracture volume to the cumulative injected volume). In the conclusion, the positive and negative features of the algorithm are discussed.

2. Statement of the problem

We consider a symmetrical vertical fracture of length 2 l propagating in infinite porous elastic medium under plain strain condition. The system of coordinates is introduced, as shown in Fig. 1. The fracture is driven by incompressible non-Newtonian power-law fluid injected at the center of the fracture along the Oy-axis.

We made the following simplifying assumptions: the fracture is planar and rectangular in the vertical section along the Oxy plane; the fracture aperture does not depend on coordinate y: $w = w(t,x)$, where t is time; the closure stress σ acts perpendicular to the fracture plane.

The rock is supposed to be linear elastic material. Fracture walls are permeable, although the influence of the pore pressure on the rock stresses is neglected. The velocity of the fluid flow through the fracture walls (the leakoff) v_l is given as a function of (t,x) or as some functional of the fluid pressure $P(t,x)$ inside the fracture. As has been done by many other authors, henceforth we will work with the net pressure $p(t,x) = P(t,x) - \sigma$ which describes the excess of the fluid pressure in the fracture over the closure stress σ.

The problem is stated as follows: given the parameters of rock (Young's modulus, Poisson's ratio, leakoff, confining stress), fluid (apparent viscosity, behavior index), and the fracturing process

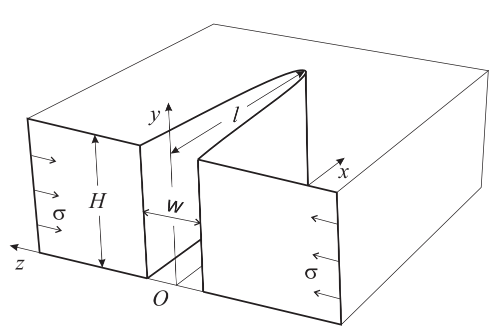

Fig. 1. The geometry of the fracture.

(flow rate), it is required to determine fracture geometry (half-length $L(t)$ and aperture $w(t,x)$) and net pressure distribution $p(t,x)$.

3. Governing equations

We assume an incompressible non-Newtonian fluid flow within the fracture of width w. Integration of the continuity equation across the section of the fracture in the direction of the Oz-axis gives the mass conservation law

$$\frac{\partial w}{\partial t} + \frac{\partial Q}{\partial x} = -v_l. \tag{1}$$

Here, Q is the flow rate through a vertical cross-section of the fracture and v_l is the leakoff speed. The expression for Q in the case of power-law non-Newtonian fluid is given by the Poiseuille law as [1]:

$$Q = -\frac{w^{(2n+1)/n}}{M^{1/n}\left|\frac{\partial p}{\partial x}\right|^{(n-1)/n}}\frac{\partial p}{\partial x}, \quad M = \frac{2^{n+1}(2n+1)^n \mu}{n^n}. \tag{2}$$

Here, μ and n are consistency and behavior indices of the fracturing fluid, respectively. Function

$$\Lambda(w, p_x) = w^{(2n+1)/n} M^{-1/n} \left|\frac{\partial p}{\partial x}\right|^{(1-n)/n}$$

will be referred to as the mobility of the fluid. Eq. (1) with the expression (2) for the flow rate yield the so-called lubrication theory equation.

4. Fracture aperture

The elastic response of the fracture walls to the net pressure in the case of two-dimensional linear elasticity theory is given by the Kolosov–Muskhelishvilli formula [16]

$$w(t,x) = \frac{4}{\pi E'} \int_0^L p(t,\xi) B(x,\xi;L)\, d\xi, \quad E' = \frac{E}{1-\nu^2}, \tag{3}$$

with the singular kernel

$$B(x,\xi;L) = \ln\left|\frac{\sqrt{L^2-x^2}+\sqrt{L^2-\xi^2}}{\sqrt{L^2-x^2}-\sqrt{L^2-\xi^2}}\right|. \tag{4}$$

Here, E and ν are Young's modulus and Poisson's ratio of the elastic media. The inverse of Eq. (3) is given by

$$p(t,x) = \frac{E'}{4\pi} \int_0^L \frac{\partial w(t,\xi)}{\partial \xi} \frac{2\xi\, d\xi}{x^2-\xi^2}. \tag{5}$$

Eq. (1) for the 1D fluid flow inside the fracture and formula (3) for the fracture disclosure form a closed system of integro-differential equations for two unknowns: p and w. Boundary conditions for this system depend on the choice of the crack propagation criterion, which is discussed in the next section.

5. Crack propagation criterion

According to classical linear elastic fracture mechanics (LEFM), the propagation of a hydraulic fracture is described in terms of the mode-I stress intensity factor [17] K_I. In the observed case of a symmetrical planar fracture in the linear elastic media, K_I can be