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Fracture-based model of periodic-arrayed indentation for rock cutting

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ABSTRACT

Periodic singular stress fields and K-dominant regions arise adjacent to sharp 90° indenter edges if an elastic substrate is subject to indentation with rigid, flat-ended and periodic indenters. The concept of indentation stress intensity factor K_{ind} is convenient to describe such a singular indentation stress field, which is mathematically similar to that of a Mode-I crack. The singular indentation stress field is sufficient to induce surface cracking even if the substrate surface is free of any micro-cracks. This surface cracking mechanism generated from the flat-ended indentation potentially can play a significant role in damage analysis of rock cutting. During the rock-cutting processing, those periodic singular-stress fields will move along the rock surface, leading to continuous rock surface fracture. In this study, a fracture mechanics model is proposed for rock breakage by using an energy-based approach. The indentation stress intensity factor K_{ind} and indentation cracking equation for rock-cutting/breakage have thus been formulated analytically.

1. Introduction

Three typical rock-cutting saws commonly used in engineering are shown in Fig. 1. The common and yet unique character is that these rock-cutting tools can be modeled by periodic rigid and flat-tipped indenters. It has been pointed out from the previous work [1–4] that a mixed-Mode singular stress field exists in an incompressible substrate at the sliding contact edge of a rigid flat-ended indenter pressing down onto the substrate. It should be emphasized that such a singular indentation stress field is sufficient to generate micro-cracks on the contact surface even if it is free of any micro-cracks. This surface cracking mechanism can potentially be utilized to assist rock-cutting, in additional to the diamond grits embedded on the cutter surface [5-7]. In this study, a fracture-based method similar to the classical fracture mechanics is proposed to formularize the cracking or indentation damage induced by the singular indentation stress field next to the corners of the periodic indenters.

2. Periodic indentation configurations

The rock-cutting model to be investigated is illustrated in Fig. 2. Periodic indenters are pressed onto the surface of half-space substrate, which occupies the region $0<x_2<h<+\infty$, $-\infty<x_1<+\infty$ and is constrained to deform in plane strain normal to the x_1-x_2 plane. The substrate is assumed to be elastically isotropic, with Young's modulus E and Poisson's ratio μ. The periodically arrayed indenters are assumed to be rigid, with contact width $2l$ and center-to-center contact spacing $2t$. Calculations for indentation stress intensity factors can be carried out based on one of the periodic Cell 1 and Cell 2 that lies between $-t/2<x_1<t/2$. Two limiting cases of friction are considered: (i) perfect smooth sliding between the indenter and substrate; and (ii) small frictional sliding contact between the indenter and substrate.

3. Asymptotic stress field in sliding contact

3.1. Boundary condition

A typical fretting contact problem of a rigid flat-ended indenter with half width a, sliding on a homogeneous, isotropic, elastic body in a half-space is shown in Fig. 3. The Cartesian coordinates (x_1, x_2) and the polar coordinates (r, θ), both with the origin at the left edge of the indenter, are selected. Normal force N and tangential force Q act on the indenter and the following normal and shear tractions along interface have been solved in closed form [4],

$$p(x_1)=-\frac{N\sin\lambda\pi}{\pi}\left(2-\frac{x_1}{a}\right)^{\lambda-1}\left(\frac{x_1}{a}\right)^{-\lambda} \quad (1)$$

and

$$q(x_1)=fp(x_1) \quad (2)$$

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Fig. 1. Three typical rock cutting saws commonly used in engineering.

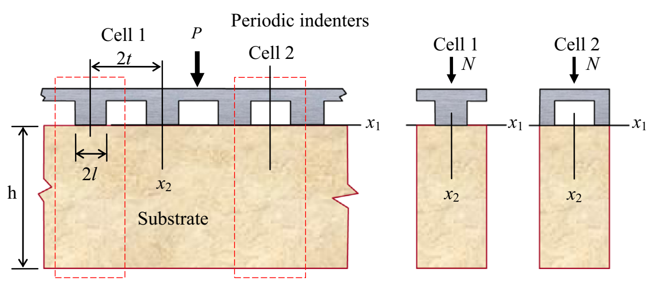

Fig. 2. The two-dimensional indentation model for rock cutting with n periodic indenters.

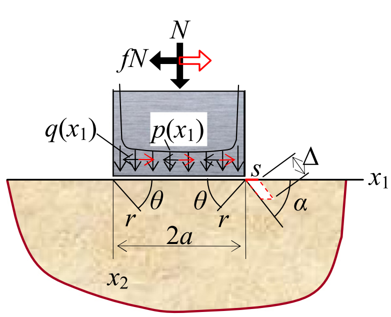

Fig. 3. Indentation configuration, integral path $s\to 0$ and $\Delta\to 0$.

where f is the coefficient of friction; λ is determined by

$$\tan\lambda\pi=\frac{2(1-\mu)}{f(1-2\mu)},\quad 0<\lambda<1. \tag{3}$$

Eq. (1) indicates that the stress state near the indenter corner may vary in the following form:

$$\sigma_{ij}=\begin{cases}0(r^{\lambda-1}), & x_1=2a\\ 0(r^{-\lambda}), & x_1=0\end{cases}\quad \text{as}\quad r\to 0. \tag{4}$$

For special cases either with $\mu=0.5$ or $f=0$, Eq. (3) leads to $\lambda=0.5$, showing the same order of stress singularity as that for a sharp crack tip.

The asymptotic stress boundary conditions of the substrate in the contact area next to the left and right corners then become

$$\sigma_{22}|_{\theta=0}=-\frac{N}{\pi\sqrt{2ar}} \tag{5}$$

$$\sigma_{21}|_{\theta=0}=\frac{fN}{\pi\sqrt{2ar}} \tag{6}$$

for the two cases of $\mu=0.5$ or $f=0$. For $\mu=0.5$, the substrate becomes incompressible.

3.2. Singular stress fields due to the normal and tangential loads

The singular stress field at the sharp edge of the contact between a rigid flat-ended indenter and substrate is known from the asymptotic contact analyses of [4]. Using the polar coordinates (r,θ) (Fig. 3) the stresses at the left corner can be found as follows due to the normal load:

$$\begin{pmatrix}\sigma_{rr}^{\mathrm{I}}\\ \sigma_{\theta\theta}^{\mathrm{I}}\\ \sigma_{r\theta}^{\mathrm{I}}\end{pmatrix}=-\frac{K_{\mathrm{I}-ind}}{\sqrt{2\pi r}}\begin{pmatrix}\cos\frac{\theta}{2}\left(1+\sin^2\frac{\theta}{2}\right)\\ \cos^3\frac{\theta}{2}\\ \sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\end{pmatrix}. \tag{7}$$

This expression indicates that the stress state for indentation is a "negative" Mode-I singular stress field for cracked solids, where

$$K_{\mathrm{I}-ind}=\frac{N}{\sqrt{\pi a}}, \tag{8}$$

which defines actually an indentation stress intensity factor. The familiar Mode-I singular stress field is obtained by removing the negative sign and changing $K_{\mathrm{I}-ind}$ into K_{I} for cracked solids with Mode-I loads. Only difference between tensile Mode-I stress field and indentation stress field is sign "−" in their equations.

Nadai [4] gave also the asymptotic stress field due to the tangential load as follows:

$$\begin{pmatrix}\sigma_{rr}^{\mathrm{II}}\\ \sigma_{\theta\theta}^{\mathrm{II}}\\ \sigma_{r\theta}^{\mathrm{II}}\end{pmatrix}=\frac{K_{\mathrm{II}-ind}}{\sqrt{2\pi r}}\begin{pmatrix}\sin\frac{\theta}{2}\left(1-3\sin^2\frac{\theta}{2}\right)\\ -3\sin\frac{\theta}{2}\cos^2\frac{\theta}{2}\\ \cos\frac{\theta}{2}\left(1-3\sin^2\frac{\theta}{2}\right)\end{pmatrix}, \tag{9}$$

where

$$K_{\mathrm{II}-ind}=fK_{\mathrm{I}-ind}. \tag{10}$$

Eq. (9) is identical to the classical Mode-II singular stress fields when $K_{\mathrm{II}-ind}=K_{\mathrm{II}}$.

3.3. Character of the stress fields

It is clear from the above discussion that the asymptotic stress field, $\sigma_{ij}=\sigma_{ij}^{I}+\sigma_{ij}^{II}$, induced by the sliding contact is a typical mixed-Mode I–II singular stress field for friction free or incompressible substrates. This singular stress field is responsible for surface crack initiation on the crack-free surface of the substrate at the contact edge. This finding is significant as it shows that singularity and distribution of the stress field induced by surface contact of a flat-ended indenter are identical to those of a mixed-Mode crack. As a result, the concepts of stress intensity factor and fracture toughness can now be introduced unambiguously into contact mechanics and associated contact damage. Therefore, Eqs. (7)–(10) represent an important advance by defining the indentation stress intensity factors, $K_{\mathrm{I}-ind}$ and $K_{\mathrm{II}-ind}$, and the K_{ind}-dominant region at the contact edge. In other words, the fracture mechanics theory, such as Griffith's criterion, is applicable in the case of the boundary fracture induced by the sliding contact. It should be pointed out that for finite boundary contact problems, Eqs. (7), (9) and (10) are still effective, for which case $K_{\mathrm{I}-ind}$ should be solved by the concerned method.