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Analytical insight into "breathing" crack-induced acoustic nonlinearity with an application to quantitative evaluation of contact cracks

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ABSTRACT

To characterize fatigue cracks, in the undersized stage in particular, preferably in a quantitative and precise manner, a two-dimensional (2D) analytical model is developed for interpreting the modulation mechanism of a "breathing" crack on guided ultrasonic waves (GUWs). In conjunction with a modal decomposition method and a variational principle-based algorithm, the model is capable of analytically depicting the propagating and evanescent waves induced owing to the interaction of probing GUWs with a "breathing" crack, and further extracting linear and nonlinear wave features (*e.g.*, reflection, transmission, mode conversion and contact acoustic nonlinearity (CAN)). With the model, a quantitative correlation between CAN embodied in acquired GUWs and crack parameters (*e.g.*, location and severity) is obtained, whereby a set of damage indices is proposed via which the severity of the crack can be evaluated quantitatively. The evaluation, in principle, does not entail a benchmarking process against baseline signals. As validation, the results obtained from the analytical model are compared with those from finite element simulation, showing good consistency. This has demonstrated accuracy of the developed analytical model in interpreting contact crack-induced CAN, and spotlighted its application to quantitative evaluation of fatigue damage.

1. Introduction

Fatigue damage, pervasive in engineering structures, has posed tremendous jeopardy to structural integrity and durability. Without timely awareness and subsequent remedial actions, fatigue damage can potentially lead to tragic consequences, incurring immense monetary wastage and even loss of life. Amongst various modalities of fatigue damage, the contact fatigue cracks are prevailing but most insidious. This sort of fatigue damage is usually initiated by deteriorative changes in material microstructures due to local accumulation of dislocations, high stress concentration, plastic deformation around inhomogeneous inclusions or other inherent imperfection, when a structure is subject to cyclic rolling and/or sliding contact loads. Progressive crack propagation from a microscopic to macroscopic degree subsequently leads to permanent damage at an observable extent [1].

The longer an engineering structure in service the more contact fatigue cracks it may develop. The presence of contact fatigue cracks in pivotal structural components (*e.g.*, aircraft engine turbine, rolling bearings or junction components in power plants) can be extraordinarily detrimental. Exemplarily, a train owned by Norfolk Southern in Columbus, the United States, detailed on July 11, 2012 [2], leading to an urgent evacuation of hundreds of residents and vast economic loss. Later investigation has revealed that the fracture of a rail section, initiated by numerous contact fatigue cracks caused by rolling train wheels after years of service of the rail section, was the culprit of this disastrous case.

To detect contact fatigue cracks at an embryo stage, qualitatively at least if not quantitatively, is an imminent task to warrant a reliable service of key engineering structures, and a rigorously defined and functionally deployed structural health monitoring (SHM) technique can accommodate such a need. Amongst existing SHM techniques [3–7] the guided ultrasonic wave (GUW)-based SHM [8–11] has proven its superb capability to strike a balance among resolution, detectability, practicality and cost, by taking advantage of appealing features of GUWs including long-range and quick probing, omnidirectional dissemination, high penetration,

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great sensitivity to damage of small dimensions, and cost-effective implementation.

The majority of existing GUW-based SHM approaches evaluate material deterioration or structural damage based on changes in linear signal features [12,13] related with present damages, such as delay in time-of-flight (ToF) [14], wave reflection and transmission [15], energy dissipation [16] and mode conversion [17]. Nevertheless, as commented earlier, the damage in real-world engineering structures usually initiates from imperceptible contact fatigue cracks that become conspicuous quite late. These fatigue cracks (with its characteristic dimension much smaller than the wavelength of the probing GUW) may not engender remarkable changes in linear GUW features. Therefore, when dealing with contact fatigue cracks, the SHM approaches relying on the use of linear GUW features may be out of their depth.

Recognition of the inefficiency of linear GUW features towards evaluating contact fatigue cracks has motivated alternative attempts to explore nonlinear features extracted from GUW signals at frequencies other than the excitation frequency of the probing GUW [18–20]. The nonlinear GUW features are commonly typified by the second-[18,21,22]/sub-harmonics [19], mixed frequency responses [23] (*e.g.*, nonlinear wave modulation spectroscopy), and shift of resonance frequency (*e.g.*, nonlinear resonant ultrasound spectroscopy) [24] to name a few, as comprehensively surveyed elsewhere [25]. Nonlinear GUW features have been proven capable of rendering enhanced detectability, sensitivity and accuracy compared with their linear counterparts. As an extra merit, nonlinear GUW features, deployed in a frequency domain, can bypass possible spatial interference from the inspected structure, therefore possessing good immunity to wave reflections and mode conversion at structural boundaries.

Various sources of nonlinearity have been scrutinized [14,26,27], on which basis the nonlinear GUW features, generated when a probing GUW interacts with a contact fatigue crack, are interpreted. The commonly recognized nonlinear sources include the contact acoustic nonlinearity (CAN), bi-linear stiffness, hysteresis, Hertzian contact, thermo-elastic coupling effect, *etc.*, as reviewed by D. Broda et al. [28]. These sources of nonlinearity jointly contribute to the manifestation of nonlinearities in captured GUW signals. In particular, the CAN has been recognized as one of the major sources to introduce nonlinearity, and has been the core of intensive research. Numerically, Wan et al. [29] studied the interaction between fundamental symmetric Lamb waves and a buried micro-crack in a thin plate using a finite-element method (FEM), showing a monotonic increasing relationship between the CAN and the length of the micro-crack. Shen and Giurgiutiu [30] adopted FEM to simulate the interaction between Lamb waves and a surface-breaking crack in a plate, giving similar results. Analytically, Solodov et al. [31] has examined the interaction between a contact crack and probing waves. By assuming a step-change in the material stiffness at the crack location, the generation of high-order harmonics induced by the crack was calibrated. Richardson [32], from an analytical perspective, explored the high-order harmonic generation, by depicting the motion of two surfaces of a crack (an unbonded interface between two media) under the modulation of traversing waves. In both studies, the probing waves were modeled as longitudinal waves, and this has essentially limited the investigation into a one-dimensional (1D) scenario. On the other hand, a two-dimensional (2D) GUW behaves differently from a 1D longitudinal wave, and it embraces both propagating and evanescent waves, each of which features multiple modes co-existing simultaneously. Superposition of individual wave modes affects the crack when a GUW traverses the crack, under which the motion of crack surfaces, in a 2D manner, is not a uniform motion throughout the entire crack surfaces as hypothesized in a 1D case. These diatheses jointly lead to a mechanism of CAN generation that is substantially different from 1D scenarios, making the existing 1D models largely fail to construe the modulation of contact fatigue cracks on propagating GUWs accurately.

With this motivation, the present work is aimed at achieving an analytical insight into the modulation of a 2D contact fatigue crack with “breathing” behaviors on GUWs. An analytical model is developed, to quantitatively interpret the underlying mechanism of CAN generation induced by a contact fatigue crack. With the model, both the propagating and evanescent waves, along with the converted modes at crack surfaces, can be depicted explicitly. An analytical prediction of the CAN generation, subjected to the severity of the crack, is obtained, on which basis a quantitative correlation between CAN embodied in acquired GUW signals and crack parameters (*e.g.*, location and severity) is ascertained. With such a correlation, the severity of a contact fatigue crack can be evaluated quantitatively.

This paper is organized as follows: modulation of a 2D “breathing” crack on propagating GUWs is modeled analytically, and detailed in the second section. The model illuminates the generation of high-order harmonics induced by the crack and predicts the crack-induced wave fields. In this section, a quantitative correlation between the nonlinear features of GUWs and crack parameters is derived; a set of linear and nonlinear indices is defined, for evaluating the severity of a contact fatigue crack. In the third section, the developed analytical model is validated against finite element simulation. Concluding remarks are presented in the last section.

2. Modulation of “breathing crack on probing GUWS – A nonlinear perspective

Consider a 2D plate-like waveguide, as illustrated schematically in Fig. 1, in which a contact crack exists along the waveguide thickness. A probing GUW is introduced into the waveguide with a transmitter (*e.g.*, a piezoelectric wafer) from the upper surface of the waveguide, left to the crack; and the probing GUW, guided by the waveguide, takes the modality of Lamb waves, to interact with the crack and accordingly produces transmitted and reflected waves that are acquired via wave receivers (or other non-contact means such as laser interferometry), respectively right and left to the crack. GUWs in the waveguide encompass multiple wave modes including symmetric and antisymmetric Lamb modes.

Limit the discussion to a lower thickness-frequency product – the highlighted region in the dispersion curves of GUWs in the waveguide shown in Fig. 2, where only the fundamental symmetric (S_0) and anti-symmetric (A_0) Lamb modes exist. For this region, the S_0 mode features a higher velocity than that of the A_0 mode, which can be beneficial to avoid the contamination from the waves reflected by structural boundaries, and therefore is selected to trigger the “breathing” behavior of the crack and introduce nonlinearity into GUWs. The interaction between the probing GUW and the contact crack embraces the following two steps in a “breathing” cycle of the crack:

(1) when the crack closes during wave compression, the propagating GUW is transmitted without inducing wave scattering; and
(2) when the crack opens during wave dilation – the case shown in Fig. 1 – the propagating GUW is partially decoupled, producing wave reflection and transmission in the waveguide.

These two steps jointly introduce the “breathing” behavior of the crack, and consequently incur wave scattering and mode conversion (*e.g.*, conversion of the S_0 mode to the A_0 mode, or generation of the first-order symmetric mode (S_1) if the frequency is