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Dynamic equivalence of ultrasonic stress wave propagation in solids

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ABSTRACT

Ultrasonic stress waves, generated during the dynamic impact on structures, were studied. A benchmark for the finite element analysis (FEA) was made to define the optimum geometrical factors, which were represented as, mesh distribution, analysis time, ultrasonic wave properties, element type, and shape to capture the dynamic phenomena compared to the theoretical exact solution. Comparison of three different dimensional finite element models was performed depending on the applied impact forces, the element size, and the structural geometry. A dynamic equivalence for these three different variables was established and found to be of a direct multiplication relation to the solid's density with the Young's modulus. The results demonstrated that the stresses in x, and y-directions, predicted by FEA simulations, matching well for the different materials under normalized time.

1. Introduction

Finite Element Analysis (FEA) is a numerical tool used independently from experimental work. Lately, FEA has been widely used in dynamic simulations to determine the stresses and deformations, the loads and forces movements, and the heat transfer, using variational matrices arrays and complex mesh diagrams [1]. Moreover, FEA has proven, in the last decade, to be a very successful tool for solving many partial differential equations and integral expression, for which no closed form solution exists. In addition, FEA minimizes the mismatch between experimental and analytical measurements; it alters the measurements to provide a closer agreement with experimental readings, using improved models which provide a close representation of the prototype problem [11]. On the other hand, FEA has a huge temptation of having a low cost relative to the experimental, which made it suitable for commercial and laboratory simulations; time dependent integration algorithms for dynamic implicit and explicit problems in FEA are some of the highly demanded applications [26]. Commonly, dynamic Loads as time dependent forces are imposed on structures by either natural earthquakes phenomena or as human activities.

As more people started to use finite element analysis and compared their results with experimentally obtained results, a normalized comparison method became essential. FEA mode synthesis with simple coordinate reduction system was analyzed [28]. The procedure was capable of analyzing complex shapes, non-linear spatial mechanisms with irregularly shaped links in high detail. Du et al. [7] started dynamic FEA simulations using 3-D elastic beam with an arbitrary moving base. They used six degrees of freedom in the finite element structural dynamic model with a pre-twisted offset mass from the elastic center base. The results provided from the FE model showed comparability to some extent, as the base motion variables used in multi-body dynamics and the fundamental elements were approximated to solve the dynamic problem of rotating beamlike structures. Camacho and Ortiz [4] developed a FE model with *Lagrangian* deformations for fracture in brittle materials, where complicated rate-dependent boundary conditions as plasticity and thermal coupling were accounted for in the calculations. The calculated fracture histories in conical, lateral and radial directions where normalized with deferent coefficients to be compared with the experiment results. Zienkiewicz, and Taylor [34] explained the success of the FEA to capture the experimental phenomena dependents on the symmetrical stability of perturbations, such as, the geometrical complex charactering, accurateness of material property, the symmetry of applied load and the optimum application of the boundary conditions. Recently the strain rate effect was introduced to the FEA through the dynamic wave properties in solids by Bonet et al. [3]. The accurateness of the time dependent properties and applied boundary conditions is the key factor of having a successful FEA, as well as the minimization of the cost, and the reducing of the formation assembly effort [27].

Thus, comparing the FEA stress results with each other becomes a significant problem due to the numerous variable factors involved in the analysis, such as, different geometries [2], applied dynamic boundary conditions [13], bi normalized terms, or the

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usage of p-method rather than k-method in solids [14]. Recent efforts in normalizing different FEA outcomes to compare the stress results in different materials and models had limited success, as the dynamic equivalence was not clearly defined for comparing stress results [10,8,32,18].

The term dynamic equivalence was first proposed by Wheeler and Mura [30]. They used variational mechanics methods for determining the different displacement mode shapes and dispersion relations for a plane time-harmonic waves, propagating through an infinitely and periodically arranged composite material. The comparison was successful due to limited variables in their problem formation. Then, Nemat-Nasser and Yamada [24] studied the harmonic stress wave propagation effect on composite layering direction; exact solutions were compared with the experimental results showing difference of less than 10%, using normalized dynamic terms. The dynamically equivalence were used to express the Saint-Venant's principle of the strip symmetric stresses [17]. As the decaying load was deduced from the average power for the dynamic fields is required for the self-equilibrium conditions. Chaix et al. [5] studied the ultrasonic wave propagation in heterogeneous solid media and compared the theoretical analysis with experimental validation normalizing results and dynamic equivalence. The theoretical results was different than the experimental for cement based media, which was associated to the poor dynamic equivalent factor used in the analysis. Zhou and McDowell [33] introduced a dynamic equivalent for the deformation of the atomistic molecular particle systems. This dynamic equivalence was attenuated using several factors as, continuum couple stress fields, continuum body force and body moment fields, continuum kinetic fields and atomic deformation constants, and the linear and angular momenta distributions. Recent analysis for an accurate dynamic equivalence under ultrasonic stress wave propagation was introduced in literature; nonetheless, with limited success [10,6,13,21,9].

In this study, a benchmark FEA model was made to define the optimum geometrical factors such as, mesh distribution, analysis time, ultrasonic wave properties, element type and shape to capture the dynamic phenomena compared with the theoretical exact solutions. The analysis started with defining the dynamic problem in two different geometries with the same boundary conditions, resulting in different stress results. Subsequently, three different 3D finite element models have been developed with the same applied impact forces, element size and numbers, and structural geometry. During the dynamic analysis was based on p-method calculations of the numerical integrated equations of motion with respect to time. Finally, the stresses of the three different developed 3D models were used to propose a dynamic equivalence, which could be therefore used for comparing stress results.

2. Theory

Ultrasonic techniques are commonly used to determine the different elastic properties of different types of materials such as, homogeneous, heterogeneous, isotropic, and anisotropic composites; as well as, detecting defects and voids within materials [20,12], as it has a wide range of applications. It is. Additionally, it is frequently adopted to measure velocity of waves, attenuation, density, and thickness. Using ultrasonic pulse echo techniques showed that the dynamic values of Young's moduli were found to be correlated with the longitudinal C_l and shear C_s wave speeds in solids, as presented in Eq. (1):

$$C_l^2 = \frac{E}{\rho}\frac{(1-\nu)}{(1+\nu)(1-2\nu)}, \quad C_s^2 = \frac{E}{\rho}\frac{1}{2(1+\nu)} \tag{1}$$

where E, ρ and ν refer to young's modulus, apparent density, and Poisson's ratio; respectively. Correlation between elastic young's modulus, bulk and shear moduli were introduced by Weng's model using variational mechanics methods [25]. It takes into account the deformations, strain and stress state of the counterparts, interfacial stresses, and the elastic energy; in determining overall moduli of the composite. The elastic energy balance of the alloy mixture was employed, in terms of average strains and stresses divided into two main parts as hydrostatic and deviatoric. The stiffness and compliance of the mixture alloy tensors were used as $L_{ijkl} = (3K, 2G)$, $M_{ijkl} = (1/3K, 1/2G)$; respectively, in terms of shear, K, bulk, G, moduli, and γ is Passion's ratio. The bulk B and shear moduli μ for the alloy two phase mixture are found as shown in Eq. (2),

$$\frac{B_c}{B_m} = 1 + \frac{V_f}{\frac{3(1-V_f)B_m}{3B_m+4\mu_m} + \frac{B_m}{B_f-B_m}}, \frac{\mu_c}{\mu_m} = 1 + \frac{V_f}{\frac{6(1-V_f)(B_m+2\mu_m)}{5(3B_m+4\mu_m)} + \frac{\mu_m}{\mu_f-\mu_m}} \tag{2}$$

where V_f is the filers volume fraction, the subscripts 'c', 'M', and 'f' denotes the alloy two phase mixture, the main matrix material and the filler properties; respectively. The α_M and β_M are main matrix constant properties, which depend directly on the matrix bulk and shear moduli. The Young's modulus is then calculated using the bulk and shear moduli as in Eq. (3),

$$B_c = \frac{E}{3(1-2\nu)}, \quad \mu_c = \frac{E}{2(1+\nu)} \tag{3}$$

Crack Initiation Toughness Average Stress Approach was analyzed by Williams in homogeneous; as well as, nonhomogeneous materials [31]. Williams used a classical Airy's stress function formulation in polar coordinates; whereas Irwin used for his analysis the complex variable method; following Westergaard, considering an infinite isotropic and homogeneous plate with infinite crack under uniform normal stresses σ and shear stresses τ. The crack tip was assumed as the origin for both Cartesian (x, y) and polar (r, θ) coordinates [31,15,16]. Williams has assumed a variable separable stress function ψ in the polar coordinates as $\psi = r^{\lambda+1}f(\theta)$. The values of the eigen parameter λ for the free edged cracked plate is as $\sin 2\pi\lambda = 0$. The stress function should satisfy the dimensional governing equation of $\nabla^2(\nabla^2\psi) = 0$ with no body forces. Where, ∇^2 is the harmonic (Laplacian) operator, $\nabla^2 = \frac{\partial}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial \theta^2}$ Also, the stress components can be defined using the stress function as presented in Eq. (4),

$$\sigma_{\theta\theta} = \frac{\partial^2\psi}{\partial r^2}, \quad \sigma_{rr} = \nabla^2\psi - \frac{\partial^2\psi}{\partial r^2}, \quad \sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right) \tag{4}$$

By replacing the Laplacian operator into the stress function the biharmonic equation becomes as in Eq. (5),

$$\begin{aligned}\nabla^2(\nabla^2\psi) = {} & (\lambda-1)(\lambda-2)r^{\lambda-3}(f''(\theta) + (\lambda+1)^2 f(\theta)) \\ & + \frac{1}{r}(\lambda-1)r^{\lambda-2}(f''(\theta) + (\lambda+1)^2 f(\theta)) + \frac{1}{r^2}r^{\lambda-1}(f''(\theta) \\ & + (\lambda+1)^2 f(\theta)) = 0\end{aligned} \tag{5}$$

The above equation is a fourth order ordinary differential equation as a function of θ. The general solution of which is presented in Eq. (6),

$$\begin{aligned}f(\theta) = {} & C_1\cos(\lambda-1)\theta + C_2\sin(\lambda-1)\theta + C_3\cos(\lambda+1)\theta \\ & + C_4\sin(\lambda+1)\theta\end{aligned} \tag{6}$$

where $C_1 - C_4$ are the unknown coefficients to be calculated from the body boundary conditions as the crack two faces are along $\theta = \pm\pi$. For stress free crack faces the stress $\sigma_{\theta\theta} = \sigma_{r\theta} = 0$, at $\theta = \pm\pi$, which leads to $f(\theta) = f(\theta) = 0$ along $\theta = \pm\pi$. By applying the boundary conditions and separating the symmetric and the anti-symmetric parts, and obtaining a non-trivial solution in each case as $\lambda = n/2$ resulting in Eq. (7) as,

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