ARTICLE IN PRESS
JID: YICAR [m5G;October 3, 2017;9:34]
Icarus 000 (2017) 1–7
Contents lists available at ScienceDirect
Icarus
journal homepage: www.elsevier.com/locate/icarus

Asteroids in three-body mean motion resonances with planets

Evgeny A. Smirnov[a,*], Ilya S. Dovgalev[b], Elena A. Popova[a]

[a] Pulkovo Observatory, Pulkovskoye chaussee 65/1, St. Petersburg, 196140, Russia
[b] 4xxi Ltd., Malaya Monetnaya 2/G, off. 403, St. Petersburg, 197101, Russia

ARTICLE INFO

Article history:
Received 15 January 2017
Revised 21 September 2017
Accepted 26 September 2017
Available online xxx

Keywords:
Resonances
Orbital
Asteroids
Dynamics
Kuiper belt

ABSTRACT

We have identified all asteroids in three-body mean-motion resonances in all possible planets configurations. The identification was done dynamically: the orbits of the asteroids were integrated for 100,000 yrs and the set of the resonant arguments was numerically analyzed. We have found that each possible planets configuration has a lot of the resonant asteroids. In total 65,972 resonant asteroids (≈14.1% of the total number of 467,303 objects from AstDyS database) have been identified.

1. Introduction

The mean motion resonances (MMRs) are playing an essential role in the dynamics of the asteroids. Besides the usual two-body MMR that involve a massive body (usually, a planet) and an asteroid, there are also so-called three-body resonances involving two planets and an asteroid. Originally the study of the asteroid three-body MMR was started with the papers about the analytical and numerical approaches to MMR (Nesvorny and Morbidelli, 1998; Murray et al., 1998). In the recent years three-body MMR are getting more attention from the researchers worldwide (Smirnov and Shevchenko, 2013; Gallardo, 2014; Milani et al., 2014; Quillen and French, 2014; Todorović and Novaković, 2015; Gallardo et al., 2016; Goździewski et al., 2016).

Thus, for example, Quillen and French (2014) investigated the resonant chains and three-body resonances in the inner Uranian satellite system. They used numerical integration of the satellites to identify the resonances. Gallardo et al. developed the semi-analytical method to estimate the strength of the three-body resonances and produced the atlas of the strongest resonances in the Solar system (Gallardo, 2014; Gallardo et al., 2016). Sekhar et al. (2016) studied the influence of three-body resonances on the meteoroid streams.

This paper extends the previous work (Smirnov and Shevchenko, 2013) in which all three-body MMR with Jupiter and Saturn have been identified (on the set of 249,567 numbered asteroids from AstDyS catalog). The goal of a current study was to find all possible three-body MMR with each combination of the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune) in the Solar system and provide full statistical results of the three-body MMRs in the Solar System.

2. Methodology

The three-body MMR is a linear combination of the mean frequencies of the orbital motion of two planets and an asteroid:

$$m_{P1}\dot{\lambda}_{P1} + m_{P2}\dot{\lambda}_{P2} + m\dot{\lambda} \approx 0, \tag{1}$$

where $\dot{\lambda}_{P1}$, $\dot{\lambda}_{P2}$, $\dot{\lambda}$ are the derivatives of mean longitudes of the first and the second planet and of an asteroid respectively and m_{P1}, m_{P2}, m — are integers.

To identify the resonance, a special parameter called “resonant argument” is introduced. It is a linear combination of the mean longitudes and the longitudes of periapsis. In the planar problem it is defined by the following formula:

$$\sigma = m_{P1}\lambda_{P1} + m_{P2}\lambda_{P2} + m\lambda + p_{P1}\varpi_{P1} + p_{P2}\varpi_{P2} + p\varpi, \tag{2}$$

where λ_{P1}, λ_{P2}, λ, ϖ_{P1}, ϖ_{P2}, ϖ are the mean longitudes and longitudes of periapsis of two planets and an asteroid respectively and m_{P1}, m_{P2}, m, p_{P1}, p_{P2}, p are integers obeying by d’Alembert rule (Morbidelli, 2002):

$$m_{P1} + m_{P2} + m + p_{P1} + p_{P2} + p = 0. \tag{3}$$

We assume the planar problem, so the longitudes of ascending nodes are not taken into account.

If the resonant argument librates (similar to the librations of a pendulum) the asteroid is in the resonance; if it circulates — then

* Corresponding author.
E-mail address: smirik@gmail.com (E.A. Smirnov).

https://doi.org/10.1016/j.icarus.2017.09.032
0019-1035/

the asteroid is out of the resonance (Chirikov, 1979; Shevchenko, 2007; Smirnov and Shevchenko, 2013).

One of the most important properties of the resonance is its *order q*, that is equal to the absolute value of the sum of integers (see Morbidelli, 2002):

$$q = |m_{P1} + m_{P2} + m|. \tag{4}$$

In the previous study, a massive identification of asteroids in the three-body resonances with Jupiter and Saturn was made (Smirnov and Shevchenko, 2013). The procedure had two steps: a formal identification that was based on the value of the semi-major axis that should be near the resonant value, and dynamical identification where the orbits of 249,567 asteroids were integrated for the period of 10^5 yrs. The total fraction of the identified resonant asteroids with order $q \le 6$ was equal to 4.4%.

However, besides the three-body resonances with Jupiter and Saturn there are also a number of other configurations. We considered all the resonances involving the 8 planets taken by pairs starting with the pair “Mercury-Venus” up to “Mercury-Neptune” and then “Venus-Earth” up to “Venus-Neptune” and so on. In our study only main subresonances ($p_{P1} = p_{P2} = 0$) were considered.

Following the algorithm introduced in Smirnov and Shevchenko (2013) we limit m_{P1}, m_{P2}, m with the following requirements:

$$q \le q_{\max} \tag{5}$$

$$|m_{P1}|,\ |m_{P2}|,\ |m| \le M_{\max}, \tag{6}$$

where $q_{\max} = 6$ and $M_{\max} = 7$.

Also we set two additional requirements:

$$m_{P1} > 0, \quad \gcd(m_{P1}, m_{P2}, m) = 1, \tag{7}$$

where gcd is the greatest common divisor.

The initial data are taken from AstDyS catalog maintained by Milani, Knežević and their coworkers (AstDyS, 2016). In this study we consider much more asteroids (467,303 instead of 249,567) than in the previous work (Smirnov and Shevchenko, 2013) because new objects have been added to the catalog. Therefore the new data for the configuration “Jupiter–Saturn” are also introduced.

2.1. Formal identification

At first stage we build so-called “identification matrix” that consists of two columns: first column gives the designation of the resonance ($m_{P1}m_{P2}m$) where P_1, P_2 are the first letters of the planets respectively (V — Venus, E — Earth, M — Mars, J — Jupiter, S — Saturn, U — Uranus, N — Neptune, except Mercury which matches with R) and all values are taken with their signs. The second column gives the resonant value of the semi-major axis calculated by the algorithm specified in Smirnov and Shevchenko (2013), with the same limitations; however, we replaced the expression for approximate estimation of the precession rate by the new formulae described in Shevchenko (2017) in order to consider both cases when the asteroid either has the inner orbit with respect to the planet or the outer orbit:

$$\dot{\varpi} = \frac{3\pi}{2}\frac{M_p}{(1+M_p)^{3/2}}\frac{a_p^2}{a^{7/2}}\left(1+\frac{3}{2}e_p^2\right), a > a_p \tag{8}$$

and

$$\dot{\varpi} = \frac{3\pi}{2}\frac{M_p a^{3/2}}{a_p^3}\left(1-e_p^2\right)^{-3/2}, a < a_p. \tag{9}$$

Here M_p, a_p, e_p are the planet mass (in Solar masses), semi-major axis (in au) and eccentricity and a is the semi-major axis of the object. In these equations we use the planet with the largest contribution to the precession rate (it is usually Jupiter, however, it can be also, e.g., Neptune in case of TNO).

Table 1
An extract from the identification matrix.

Resonance	a_{res} (au)
5J − 2S − 2	3.1753
4J − 6U − 1	2.4192
4E − 7M − 1	2.3457
6S − 3N − 1	3.0764
1M − 3J − 1	2.3425

Unlike before now we have to show directly the planets involved in the resonance because the designation without planets' identifiers is not unique anymore (e.g. 2−3+1 could be resonance with Jupiter and Saturn or Venus and Earth). The new format is proposed: 2V−3E+1 means the three-body resonance with Venus and Earth where integers are equal to 2, −3 and 1 for Venus, Earth and asteroid respectively, 5J−2S−2 — three-body resonance with Jupiter and Saturn where the integers are equal to 5, −2, −2 for Jupiter, Saturn and asteroid respectively. The planets in the designation are ordered by the distance from the Sun.

An example of the identification matrix (a few rows) is presented in Table 1. The full identification matrices are available at the following link: https://goo.gl/T38gFx.

2.2. Dynamical identification

The algorithm of the dynamical identification is based on a procedure described in Smirnov and Shevchenko (2013), however, we introduce some changes:

1. each asteroid from the set of 467,303 objects is integrated for the period of 10^5 yrs;
2. mercury6 integrator is used (Chambers, 1999);
3. all perturbations from the planets and Pluto are taken into account;
4. the output interval is set to 10 yrs.

For each asteroid, we calculate a set of possible resonances based on the closeness of the asteroid's semi-major axis to the resonant value of the semi-major axis for the given resonance (that we have computed in the previous step). The resonant argument is analyzed automatically to identify the libration/circulation. The identification procedure for a chosen asteroid is described below.

1. We create the time series of the semi-major axis and the resonant argument, the time space between the points is 10 years, the series contain in common 10,001 data points from 0 to 100,000 years. After that we build the periodograms using the FFT method (Jenkins and Watts, 1969), the resonant argument σ is first transformed to $z = e^{i\sigma}$ ($i^2 = -1$).
2. To get smoothed version of the time series we apply the digital low-pass filter A on z and on the semi-major axis (Nesvorny and Ferraz-Mello, 1997; Quinn et al., 1991). In our case the filter eliminates the components with period less than ≈300 years.
3. We run over the original time series of the resonant argument from the starting point checking whether the difference between the current point and the starting point is more than 2π. If so, we start the local classification of this section by a several steps:
   - If the length of the section is less than a certain “circulation” parameter (signed as *cp*, chosen ≈2000 years, similar parameter has been used in Nesvorny and Morbidelli, 1998), this section is automatically qualified as circulating, the following step is skipped and a new section is being started from the current point.

Please cite this article as: E.A. Smirnov et al., Asteroids in three-body mean motion resonances with planets, Icarus (2017), https://doi.org/10.1016/j.icarus.2017.09.032