New Astronomy 61 (2018) 100–103

Contents lists available at ScienceDirect

New Astronomy

journal homepage: www.elsevier.com/locate/newast

# Possible relation between pulsar rotation and evolution of magnetic inclination

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ARTICLE INFO

*Keywords:*
Pulsars
General

ABSTRACT

The pulsar timing is observed to be different from predicted by a simple magnetic dipole radiation. We choose eight pulsars whose braking index was reliably determined. Assuming the smaller values of braking index are dominated by the secular evolution of the magnetic inclination, we calculate the increasing rate of the magnetic inclination for each pulsar. We find a possible relation between the rotation frequency of each pulsar and the inferred evolution of the magnetic inclination. Due to the model-dependent fit of the magnetic inclination and other effects, more observational indicators for the change rate of magnetic inclination are needed to test the relation.

## 1. Introduction

Pulsars are highly magnetized rotating neutron stars with beams of emission that are observed as pulses. The slowdown of rotation is manifest in the actual arrival times of pulses. Usually the loss of kinetic rotational energy is understood to be transformed to electromagnetic radiation. In the idealized model, neutron stars slow down because of the generation of magnetic dipole radiation. In this case, the slowdown of frequency is given by

$$\dot{\nu} = -\frac{8\pi^2}{3c^3}\frac{M^2 sin^2\alpha}{I}\nu^3 \tag{1}$$

where $M$ is the magnetic dipole moment, $I$ the moment of inertia, $\alpha$ the inclination angle between the magnetic and rotation axis. In the canonical model $M, I$ and $\alpha$ are constants, and a dependency $\dot{\nu} \propto \nu^3$ for the slowdown of pulsars would be expected.

However the observed dependence of $\dot{\nu}$ on $\nu$ differs from predicted by the magnetic dipolar radiation. The power-law slowdown can be written in a more general form

$$\dot{\nu} \propto \nu^n \tag{2}$$

where $n$ is known as the braking index. Reliable values of $n$ were obtained for a few pulsars (see Table 1). Most of the braking index values are less than 3 except for PSR J1640-4631 which was recently measured to be 3.15 (Archibald et al. 2016).

The discrepancy of observed braking index challenge the canonical model, and different approaches have been tried to improve it (Blandford and Romani, 1988; Allen and Horvath, 1997; Melatos, 1997; Contopoulos and Spitkovsky, 2006; Magalhaes et al., 2012; Kou and Tong, 2015). One way is to consider the time evolution of the three constants in equation (1) $M, I$ and $\alpha$. The evolution of magnetic inclination is most intriguing since the report of an increasing $\alpha$ in Crab pulsar though very slow (Lyne et al., 2013). Most excitingly, the observed braking index value of 2.51 (Lyne et al., 1993) can be remarkably explained if the departure from classic slowdown is solely attributed to a secular change in $\alpha$. Inspired by this success, we tried to interpret the small values of braking index for the eight pulsars in Table 1 being dominated by a secular anti-alignment between magnetic and rotation axis. From equation (1), $\dot{\nu} \propto \nu^3 sin^2\alpha$. So the modified braking index is given by

$$n = 3 + 2\nu/\dot{\nu} \times \dot{\alpha}/tan\alpha \tag{3}$$

If the magnetic inclination angle is well constrained, we can derive its change rate $\dot{\alpha}$ from this equation. One method to constrain the viewing geometry is to fit the observed polarization position angle swing by the Rotating Vector Model (Radhakrishnan and Cooke, 1969). However for the pulsars in Table 1 no reliable parameters of viewing geometry are available due to extreme uncertainties. So the $\alpha$ values adopted in this work depend on different models. Most of them were derived from the fit of high-energy light curves.

## 2. Review of model dependent $\alpha$

For the Crab pulsar the shape of the beam has been modeled by several authors (Dyks and Rudak, 2003; Harding et al., 2008; Watters et al., 2009; Du et al., 2012), and the estimated range of $\alpha$ is between 45° and 70°. A model of high-energy light curves whose double peaks are supposed to arise from a crossing two caustics and associated with

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https://doi.org/10.1016/j.newast.2017.12.006
Received 27 September 2017; Received in revised form 2 December 2017; Accepted 26 December 2017

**Table 1**
Rotation frequencies for pulsars with known braking indices. Time derivatives are denoted by a dot.

| PSR | n | $\nu(s^{-1})$ | $\dot{\nu}(\times 10^{-11}s^{-2})$ | reference |
|---|---|---|---|---|
| B0531 + 21(Crab) | 2.51(1) | 30.2254371 | −38.6228 | Lyne et al. (1993) |
| B0540−69 | 2.14(1) | 19.8344965 | −18.8384 | Livingstone et al. (2007) |
| B0833−45(Vela) | 1.4(2) | 11.2 | −1.57 | Lyne et al. (1996) |
| J1119−6127 | 2.684(2) | 2.4512027814 | −2.415507 | Weltevrede et al. (2011) |
| B1509−58 | 2.839(1) | 6.633598804 | −6.75801754 | Livingstone et al. (2007) |
| J1734−3333 | 0.9(2) | 0.855182761 | −0.166702 | Espinoza et al. (2011) |
| J1833−1034 | 1.857(1) | 16.15935712 | −5.275113001 | Roy et al. (2012) |
| J1846−0258 | 2.65(1) | 3.0782148166 | −6.71562 | Livingstone et al. (2007) |
| J1640−4631 | 3.15(3) | 4.84341082106 | −2.280775 | Archibald et al. (2016) |

different magnetic poles produces generic features consistent with observed pulsar light curves, and particularly with $\alpha = 60^\circ$ the model is well suited for the light curve of the Crab pulsar and $\alpha = 70^\circ$ for the Vela pulsar (Dyks and Rudak, 2003). A 3D model of emission from optical to gamma-ray band originating from the high-altitude slot gap accelerator can reasonably well reproduce the Crab pulsar profile and spectrum with $\alpha = 45^\circ$(Harding et al., 2008). A simulation of the beaming pattern and light curves for young spin-powered pulsars leads to an estimate of $\alpha$ large as 70° with an outer gap geometry and a range between 55° and 60° with two pole caustic (Watters et al., 2009). In the same simulation $\alpha$ for the Vela pulsar is estimated to be 75° configured with outer gap and between 62° and 68° with two pole caustic. A fit of the phase-averaged spectrum and phase-resolved spectra of the Crab pulsar, which was best modeled by the annular gap emission shows an inclination angle of 45° (Du et al., 2012).

For PSR B0540-69, based on the 3D outer magnetosphere model of pulsars (Cheng et al., 2000) the calculated light curve and spectrum assuming a magnetic inclination of 50° are similar to the observed data (Zhang and Cheng, 2000). Using the same model, a magnetic inclination of 60° is consistent with the observed data for PSR B1509-58. In a later simulation (Takata and Chang, 2007) a smaller inclination angle $\alpha = 30^\circ$ for PSR B0540-69 was adopted to reproduce a pulse profile more consistent with observation.

For the Vela pulsar a fit of the phase-averaged spectrum whose gamma-ray emission was modeled by photon-photon pair process in the outer gap gives $\alpha = 71 \pm 1.\ 1^\circ$ (Li et al., 2013). In the same case $\alpha = 70^\circ$ was estimated for PSR J1833-1034 by calculating the best exponential cutoff power-law fit since the observed spectral data were not available. It can be seen the observed spectrum for this pulsar cannot be reproduced well in this model. Another model of the retarded vacuum dipole field in conjunction with standard outer gap emission geometry was applied to fit the gamma-ray light curve of the Vela pulsar and present an optimal solution $\alpha = 78 \pm 1^\circ$ (Barnard et al., 2016).

The polarization profiles are available for PSR J1119-6127 and PSR B1509-58 (Rookyard et al., 2015a). The fit of the position angles to the Rotating Vector Model constrains the viewing geometry. For PSR J1119-6127 two favored solutions $\alpha = 9.\ 1^\circ$ or 6.9° were given corresponding to the rotating radio transients like components outside and inside the open field line region respectively(values of $\alpha > 90^\circ$ have been mapped into $0 < \alpha < 90^\circ$). For PSR B1509-58 the favored solution was $\alpha = 13.\ 7^\circ$. After correcting the values of $\alpha$ based on an intrinsic sinusoidal distribution, the favored solution for PSR J1119-6127 was $\alpha = 21^\circ$ or 16° and for PSR B1509-58 $\alpha = 30^\circ$ (Rookyard et al., 2015b). Another fit of the observed gamma-ray spectrum and energy dependent light curves for PSR B1509-58 under the outer gap model found $\alpha = 20^\circ$ (Wang et al., 2013).

For the GeV-quite soft gamma-ray PSR J1846-0258, the soft spectrum and single-peak light curve could be well explained by a model suggesting the emissions produced via synchrotron radiation of the electron-positron pairs which are created by the inward gamma rays interacting with the strong magnetic field near the polar cap region with the viewing geometry of $\alpha = 10^\circ$ (Wang et al., 2014). For high-magnetic radio pulsar PSR J1734-3333, the calculations of magnetic inclination suggest $\alpha = 6^\circ$ if the line of sight passes through the center of the emission cone and $\alpha = 21^\circ$ if polarization data is taken into account (Nikitina and Malov, 2017). In the same calculation, we can see for PSR J1119-6127 $\alpha \sim 6^\circ$ and 17° via the two methods respectively, and for PSR B1509-58 $\alpha \sim 3^\circ$ and 10°.

PSR J1640-4631 is the only pulsar with a reliable braking index larger than three. The braking index 3.15 ± 0.03 (Archibald et al. 2016) cannot be attributed to the anti-alignment of magnetic axis. It can be explained by the plasma-filled magnetosphere model (Spitkovsky, 2006; Philippov et al., 2014) for two different inclination angles, 18.5° ± 3° and 56° ± 4° (Eksi and Andac, 2016). The smaller value is favored by the single-peak pulse profile. The rate of decrease of the inclination angle was found to be $(-0.23 \pm 0.05)^\circ$ *century* $^{-1}$.

## 3. Possible relation between $\dot{\alpha}$ and $\nu$

For each of the nine pulsars differently favored values of the magnetic inclination angle have been reported by several authors as listed in Table 2. ('The distribution of $\alpha$ cited here is plotted in Fig. 1 against the spin frequencies' is deleted) The orientation of magnetic axis for different pulsars range from nearly alignment with the rotation axis to

**Table 2**
The different values of magnetic inclination depend on different models. The corresponding change rate of magnetic inclination is supposed to produce the braking index smaller than three. Note for PSR J1640-4631 the value of $\dot{\alpha}$ is predicted by the plasma-filled magnetosphere model in the reference.

| PSR | $\alpha$(degree) | $\dot{\alpha}(10^{-12}$rad/s) | Reference |
|---|---|---|---|
| B0531 + 21(Crab) | 60 | 5.35 | Dyks and Rudak (2003) |
| | 45 | 3.089 | Harding et al. (2008);<br>Du et al. (2012) |
| | 70 | 8.486 | Watters et al. (2009) |
| | 55–60 | 4.411−5.35 | Watters et al. (2009) |
| B0540-69 | 50 | 4.853 | Zhang and Cheng (2000) |
| | 30 | 2.351 | Takata and Chang (2007) |
| B0833-45(Vela) | 70 | 3.076 | Dyks and Rudak (2003) |
| | 75 | 4.178 | Watters et al. (2009) |
| | 62–68 | 2.106−2.771 | Watters et al. (2009) |
| | 69.9–72.1 | 3.059−3.466 | Li et al. (2013) |
| | 77–79 | 4.849−5.76 | Barnard et al. (2016) |
| J1119-6127 | 9.1 | 0.2494 | Rookyard et al. (2015a) |
| | 6.9 | 0.1884 | Rookyard et al. (2015a) |
| | 21 | 0.5977 | Rookyard et al. (2015b) |
| | 16 | 0.4465 | Rookyard et al. (2015b) |
| | 6 | 0.1636 | Nikitina and Malov (2017) |
| | 17 | 0.476 | Nikitina and Malov (2017) |
| B1509-58 | 60 | 1.412 | Zhang and Cheng (2000) |
| | 13.7 | 0.199 | Rookyard et al. (2015a) |
| | 30 | 0.471 | Rookyard et al. (2015b) |
| | 20 | 0.297 | Wang et al. (2013) |
| | 3 | 0.043 | Nikitina and Malov (2017) |
| | 10 | 0.144 | Nikitina and Malov (2017) |
| J1734-3333 | 6 | 0.2151 | Nikitina and Malov (2017) |
| | 21 | 0.7857 | Nikitina and Malov (2017) |
| J1833-1034 | 70 | 5.1258 | Li et al. (2013) |
| J1846-0258 | 10 | 0.67157 | Wang et al. (2014) |
| J1640-4631 | 18.5 ± 3 | $-(1.3 \pm 0.3)$ | Eksi and Andac (2016) |