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Phase space deformations in phantom cosmology

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ABSTRACT

We discuss the physical consequences of general phase space deformations on the minisuperspace of phantom cosmology. Based on the principle of physically equivalent descriptions in the deformed theory, we investigate for what values of the deformation parameters the arising descriptions are physically equivalent. We also construct and solve the quantum model and derive the semiclassical dynamics.

1. Introduction

One of the most intriguing aspects in physics, is the current acceleration of the Universe. Is it a consequence of modifications to general relativity (GR) or is it a new kind of matter that drives this acceleration?. Although having some theoretical problems, the best answer to this question is the cosmological constant Λ. An alternative candidate that has been successful for the description of dark energy is the scalar field [1–6]. In particular, scalar fields with a negative kinetic term have been considered in the literature. This type of fields are known as Phantom fields. They have some interesting properties that allows them to be considered as a strange but viable matter which could be relevant in the evolution of the Universe [7]. In particular, a phantom field provides an effective negative pressure and a repulsive effect on the matter content of the Universe which in the long term could be responsible for the current accelerated expansion [8–12]. Therefore it has been considered as the matter source of the late time accelerated expansion of the Universe [13–15].

When considering the spacetime structure of the Universe it is usually done in reference to GR. But when regarding the micro structure of spacetime we do not have a universally accepted quantum theory of gravity. Although there are several candidates, noncommutativity has been considered as an alternative to understand the small scale structure of the Universe and help in the construction of quantum theory of gravity. For this reason, noncommutative versions of gravity have been constructed [16–19]. Noncommutativity is usually believed to be present near Planck's scale and is consistent with a discrete nature of spacetime. Motivated by this idea, it is justified to consider an inherently noncommutative spacetime at the early ages of the universe. Directly using noncommutative gravity is quite difficult, this is a consequence of the highly nonlinear character of these theories [16–19]. Fortunately there is an alternative, in [20] the authors introduce the effects of noncommutativity using the Moyal product of functions on the Wheeler–DeWitt (WDW) equation.

The effects of the noncommutative deformation at the classical level was studied by a WKB approximation of noncommutative quantum model [21] and also by modifying the Poisson algebra [22,23]. More general minisuperspace deformations have been done in connection with Λ [24–29]. Phase space deformations give rise to two physically nonequivalent descriptions, the *"C-frame"* based on the original variables but with a modified interaction and the *"NC-frame"* frame based on the deformed variables but with the original interaction. Given this ambiguity, in [29] a principle was proposed to restrict the value of the deformation parameters in order to make both descriptions physically equivalent.

In this work we consider a Friedmann–Robertson–Walker (FRW) cosmological model coupled to a phantom scalar field and study the physical consequences of introducing general phase space deformations on the minisuperspace of the theory. We will study both, the classical and quantum models and we also find that the semiclassical approximation of the deformed quantum model agrees with the classical model. The deformation parameter space is determined by considering the principle of physically equivalent frames [29].

The paper is organized as follows. In Section 2, the commutative model is presented. In Section 3, the minisuperspace phase space deformation is implemented and the dynamics in the two frames is obtained. Also the deformation parameters are constrained by

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imposing physical equivalence between the *"C-frame"* and the *"NC-frame"*. The quantum analysis is done in Section 4, we find the solution to the deformed WDW equation and fix the parameters in the deformation in order to make the quantization possible. We also show that the classical paths arise from the semiclassical approximation of the WDW equation obtained from the deformed Hamiltonian. Finally, Section 5 is devoted for concluding remarks.

2. Phantom field model

We start with the flat FRW metric

$$ds^2 = -N^2(t)dt^2 + a^2(t)[dr^2 + r^2 d\Omega] \tag{1}$$

$a(t)$ corresponds to the scale factor and $N(t)$ is lapse function. In this background the action of a minimally coupled phantom scalar field $\varphi(t)$ with constant potential is

$$S = \int dt \left\{ -\frac{3a\dot{a}^2}{N} - a^3\left(\frac{\dot{\varphi}^2}{2N} + N\Lambda\right)\right\}, \tag{2}$$

where we have set the units so $8\pi G = 1$. The minus sign in the kinetic term of the scalar action is the difference between the usual scalar field and the phantom scalar field [12]. The canonical Hamiltonian derived from Eq. (2) is

$$-N\left[\frac{P_a^2}{12a} + \frac{P_\varphi^2}{2a^3} - a^3\Lambda\right]. \tag{3}$$

With the following change of variables

$$x = \mu^{-1}a^{3/2}\sin(\mu\varphi), \tag{4}$$
$$y = \mu^{-1}a^{3/2}\cos(\mu\varphi),$$

and $\mu = \sqrt{3/8}$. The Hamiltonian Eq. (3) can be rewritten as a sum of two harmonic oscillators

$$H = N\left(\frac{1}{2}P_x^2 + \frac{\omega^2}{2}x^2\right) + N\left(\frac{1}{2}P_y^2 + \frac{\omega^2}{2}y^2\right), \tag{5}$$

where $\omega^2 = -\frac{3}{4}\Lambda$. When one considers the usual scalar field, the Hamiltonian is transformed to a "ghost oscillator" which is simply a difference of two harmonic oscillators [26,29].

3. Deformed phase space model

There exist different approaches to incorporate noncommutativity into physical theories. Particularly, in cosmology there is a broadly explored path to study noncommutativity [20], where noncommutativity is realized in the so called minisuperspace variables. We will follow a deformed phase space approach. The deformation is introduced by the Moyal bracket $\{f, g\}_\alpha = f\star_\alpha g - g\star_\alpha f$. By substituting the usual product with the Moyal product $(f \star g)(x) = \exp\left[\frac{1}{2}\alpha^{ab}\partial_a^{(1)}\partial_b^{(2)}\right] f(x_1)g(x_2)|_{x_1=x_2=x}$ such that

$$\alpha = \begin{pmatrix} \theta_{ij} & \delta_{ij} + \sigma_{ij} \\ -\delta_{ij} - \sigma_{ij} & \beta_{ij} \end{pmatrix}. \tag{6}$$

The 2×2 antisymmetric matrices θ_{ij} and β_{ij} represent the noncommutativity in the coordinates and momenta respectively. The α deformed algebra becomes

$$\{x_i, x_j\}_\alpha = \theta_{ij}, \{x_i, p_j\}_\alpha = \delta_{ij} + \sigma_{ij}, \ \{p_i, p_j\}_\alpha = \beta_{ij}. \tag{7}$$

In this work we use the particular deformations, $\theta_{ij} = -\theta\epsilon_{ij}$ and $\beta_{ij} = \beta\epsilon_{ij}$.

There is an alternative to derive a similar algebra to Eq. (7). For this we follow the procedure given in [24,26]. Start with the transformation

$$\widehat{x} = x + \frac{\theta}{2}P_y, \qquad \widehat{y} = y - \frac{\theta}{2}P_x,$$
$$\widehat{P}_x = P_x - \frac{\beta}{2}y, \qquad \widehat{P}_y = P_y + \frac{\beta}{2}x, \tag{8}$$

on the classical phase space variables $\{x, y, P_x, P_y\}$, these are the variables that satisfy the usual Poisson algebra. The new variables satisfy a deformed algebra

$$\{\widehat{y}, \widehat{x}\} = \theta, \ \{\widehat{x}, \widehat{P}_x\} = \{\widehat{y}, \widehat{P}_y\} = 1 + \sigma, \ \{\widehat{P}_y, \widehat{P}_x\} = \beta, \tag{9}$$

where $\sigma = \theta\beta/4$. Furthermore, as in [24,26], we assume that the deformed variables satisfy the same relations as their commutative counterparts. The resulting algebra is the same, but the Poisson bracket is different in the two algebras. In Eq. (7), the brackets are the α deformed ones related to the Moyal product, for the other algebra the brackets are the usual Poisson brackets.

To construct the deformed theory, we start with a Hamiltonian which is formally analogous to Eq. (5) but constructed with the variables that obey the modified algebra Eq. (9), this gives the deformed Hamiltonian

$$H_{nc} = \frac{1}{2}\left[(P_x^2 + P_y^2) + \ell^2(xP_y - yP_x) + \widetilde{\omega}^2(x^2 + y^2)\right], \tag{10}$$

where ℓ^2 and $\widetilde{\omega}^2$ are given by

$$\ell^2 = \frac{\beta + \omega^2\theta}{1 + \frac{\omega^2\theta^2}{4}}, \qquad \widetilde{\omega}^2 = \frac{\omega^2 + \frac{\beta^2}{4}}{1 + \frac{\omega^2\theta^2}{4}}. \tag{11}$$

There is a significant difference between the transformed Hamiltonian of the scalar field cosmology model [26] and Eq. (10), the deformed Hamiltonian of the phantom cosmology model. Unlike the scalar field case, the crossed term involving position and momentum in the phantom model, corresponds to an angular momentum term. To understand the physics of the deformation, it is necessary to remember that the deformation Eq. (8) defines two physical nonequivalent descriptions, the *"C-frame"* where the effects of the deformation are interpreted as a commutative space (x, y) but with modification of the original interaction and the *"NC-frame"* where we work with the deformed variables $(\widehat{x}, \widehat{y})$ and the original interaction. In general, the dynamics in the two frames is different but the physical interpretation in the *"C-frame"* is easier. In this frame we can interpret the deformed model as a bidimensional harmonic oscillator and the minisuperspace deformation comes into play as an angular momentum term in the Hamiltonian. For this reason we will do the calculations in the *"C-frame"*.

We obtain the equations of motion from the Hamiltonian Eq. (10), which in the (x, y) variables are given by

$$\dot{x} = P_x - \frac{1}{2}\ell^2 y, \qquad \dot{y} = P_y + \frac{1}{2}\ell^2 x, \tag{12}$$
$$\dot{P}_x = -\frac{1}{2}\ell^2 P_y - \widetilde{\omega}^2 x, \qquad \dot{P}_y = \frac{1}{2}\ell^2 P_x - \widetilde{\omega}^2 y,$$

and get

$$\ddot{x} + \ell^2\dot{y} + \left(\widetilde{\omega}^2 - \frac{\ell^4}{4}\right)x = 0, \tag{13}$$
$$\ddot{y} - \ell^2\dot{x} + \left(\widetilde{\omega}^2 - \frac{\ell^4}{4}\right)y = 0.$$

With the transformation $z = x + iy$ we can easily solve Eq. (13). We have three different solutions depending on the sign of $\widetilde{\omega}^2$. For $\widetilde{\omega}^2 > 0$, we get

$$x(t) = A_1\cos[(\ell^2/2 + |\widetilde{\omega}|)t] + B_1\cos[(\ell^2/2 - |\widetilde{\omega}|)t], \tag{14}$$
$$y(t) = A_1\sin[(\ell^2/2 + |\widetilde{\omega}|)t] + B_1\sin[(\ell^2/2 - |\widetilde{\omega}|)t],$$