ARTICLE IN PRESS

Planetary and Space Science xxx (2017) 1–7

Contents lists available at ScienceDirect

Planetary and Space Science

journal homepage: www.elsevier.com/locate/pss

Trilogy, a planetary geodesy mission concept for measuring the expansion of the solar system

David E. Smith[a,*], Maria T. Zuber[a], Erwan Mazarico[b], Antonio Genova[a], Gregory A. Neumann[b], Xiaoli Sun[b], Mark H. Torrence[c], Dan-dan Mao[d]

[a] Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[b] Solar System Exploration Division, NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA
[c] Stinger Ghaffarian Technologies, Inc., Greenbelt, MD 20770, USA
[d] Sigma Space Corporation, Lanham, MD 20706, USA

ABSTRACT

The scale of the solar system is slowly changing, likely increasing as a result of solar mass loss, with additional change possible if there is a secular variation of the gravitational constant, *G*. The measurement of the change of scale could provide insight into the past and the future of the solar system, and in addition a better understanding of planetary motion and fundamental physics. Estimates for the expansion of the scale of the solar system are of order 1.5 cm year^{-1} AU^{-1}, which over several years is an observable quantity with present-day laser ranging systems. This estimate suggests that laser measurements between planets could provide an accurate estimate of the solar system expansion rate. We examine distance measurements between three bodies in the inner solar system – Earth's Moon, Mars and Venus – and outline a mission concept for making the measurements. The concept involves placing spacecraft that carry laser ranging transponders in orbit around each body and measuring the distances between the three spacecraft over a period of several years. The analysis of these range measurements would allow the co-estimation of the spacecraft orbit, planetary ephemerides, other geophysical parameters related to the constitution and dynamics of the central bodies, and key geodetic parameters related to the solar system expansion, the Sun, and theoretical physics.

1. Introduction

The motions of the planets are a response to the totality of forces within the solar system and the fundamental laws of physics. The dominant force is that of gravity, principally of the Sun but also the gravitational interaction of each solar system body with every other body. Observations of the motions of the planets in the solar system over an extended period of time have embodied in them both the changes in our understanding of the laws of physics and the changes to our knowledge of the individual bodies within the solar system. (Folkner et al., 2014; Williams et al., 1996; Genova et al., 2017). Of major interest are the Sun's gravity field, which governs the scale of the solar system, and the laws of both Newtonian and relativistic physics that determine the dynamical evolution of solar system.

The mass of the Sun is believed to be slowly decreasing due to the conversion of hydrogen to helium within the deep solar interior, in which 2.9% of a proton mass is lost in the reaction (e.g., Sackmann et al., 1993; Noerdlinger, 2008). This lost mass is converted into energy and radiated from the sun as electromagnetic (E-M) and particle radiation in the solar wind and coronal mass ejections. Changes in solar radiation output and magnetic polarity, the former known to occur with periods of 11 and 22 years, collectively suggest that the loss of mass may not be constant. The decrease in the solar mass results in decreased gravitational attraction of planetary bodies in heliocentric orbit, resulting in an increase in orbital distances as the Sun progresses through the main sequence phase of its evolution. The measurement of changes in the orbits of the planets may thus provide insight into the nature of the Sun's deep interior processes. Other physical effects also affect the orbits of planets, and these must be understood in order to isolate the solar contribution.

2. Science background

Based upon the flux of radiation emitted by the Sun and the mass in the solar wind, the fraction of solar mass loss is of order (Zuber et al., 2017)

$$M - dot/M = -1 \times \ 10^{-13} yr^{-1} \tag{1}$$

where *M-dot* is the time derivative of the solar mass, *M*.

Despite the fact that the rate of the Sun's mass loss is small, its effect

* Corresponding author.
E-mail address: smithde@mit.edu (D.E. Smith).

https://doi.org/10.1016/j.pss.2018.02.003
Received 2 September 2017; Received in revised form 20 December 2017; Accepted 6 February 2018
Available online xxxx
0032-0633/

on the solar system is not insignificant. The major axis of a planetary orbit is inversely proportional to *GM*, the product of the solar mass, *M* and the gravitational constant, *G*. It has also been suggested that the gravitational constant could be changing, in which case it would also affect the scale of planetary orbits. Estimates of the rate of change of *G* have been derived from lunar laser ranging and analysis of pulsar timing. Some recent results are summarized in Table 1, but in all cases uncertainties are such that the results are consistent with a *G-dot* of zero. However, a change in *G* cannot be separated from a change in *M* in the motion of the planets about the sun but can be obtained independently, at least in principle, from changes in the Earth-Moon distance.

From Kepler's third law and conservation of angular momentum (for circular orbit) we see that

$$2\delta n/n + 3\delta a/a = \delta\mu/\mu, \qquad \delta n/n + 2\delta a/a = 0, \tag{2}$$

where $\mu = GM$, n is the mean angular motion, a is the semi-major axis, δ is change, and

$$\delta\mu/\mu = -\delta a/a, \ \ and \ \ \delta\mu/\mu = ½.\delta n/n, \tag{3}$$

which indicates that a decrease in M yields a linear increase in a and a linear decrease in n.

For $M\text{-}dot/M = -1 \times 10^{-13}\ \mathrm{yr}^{-1}$, the Earth's orbit increases by $\sim 1.5\ \mathrm{cm\,yr}^{-1}$ and the orbital velocity changes by $\sim$ -9.4 $\mathrm{cm\,yr}^{-1}$. This calculation assumes that angular momentum is conserved, but we recognize that an unknown small amount of momentum is lost to the solar wind and is ignored in this work.

Fig. 1 shows the effect of a change in *GM*, on planetary orbit size and orbital velocity. Although both distance and angular velocity scale linearly, orbital velocity does not scale linearly with distance from the sun and thus shows a larger effect for the inner planets than the outer planets. Note that as the orbital radius increases the angular velocity decreases, so the planet falls behind its original path in addition to moving outward.

3. Changes in the distances between planets

We can estimate the magnitude of the change in a planet's position from a changing solar *GM* by propagating the positions of the planets. Fig. 2 shows the direct effect in distance between the four inner planets, Mercury, Venus, Earth and Mars for a $GM\text{-}dot/GM = 1 \times 10^{-13}\ \mathrm{yr}^{-1}$ over a four-year period.

The oscillations in the distances are at the synodic periods of planet pairs, and the relative distance anomaly steadily grows quadratically in amplitude over time as the two planets move further from their original trajectories (this is shown more clearly in Fig. 5 where the time base is 3 000 days). Mercury has the largest orbital velocity and causes the largest signal over the time span, with a total amplitude of more than 3–4 m.

Measuring any one of the relative distances over time in Fig. 2 can provide an estimate of the change in *GM* as has been demonstrated by Genova et al. (2017), who obtained $(-6.13 \pm 1.41) \times 10^{-14}$ from analysis of range and range-rate tracking data of the MESSENGER spacecraft in Mercury orbit over a 4-year period, 2011–2015. The ability to extract the signal of a changing solar *GM* is largely the result of the unique signature in the range measurements between Earth and Mercury, and the quadratic nature of the increase in magnitude seen in Fig. 2 and more clearly in Fig. 5. Correlations with other parameters, such as the state

Table 1
Recent estimates of the time derivative in the gravitational constant, *G*.

Data Set	Gdot/G	Reference
Pulsar timing	$(-0.6 \pm 1.1) \times 10^{-12}\ \mathrm{y}^{-1}$	Zhu et al. (2015)
Lunar laser ranging	$(1 \pm 2.5) \times 10^{-13}\ \mathrm{y}^{-1}$	Müller et al. (2014)
Lunar laser ranging	$(-0.7 \pm 3.8) \times 10^{-13}\ \mathrm{y}^{-1}$	Hofmann et al. (2010)
MESSENGER mission	$<2 \times 10^{-14}\ \mathrm{y}^{-1}$	Genova et al. (2017)

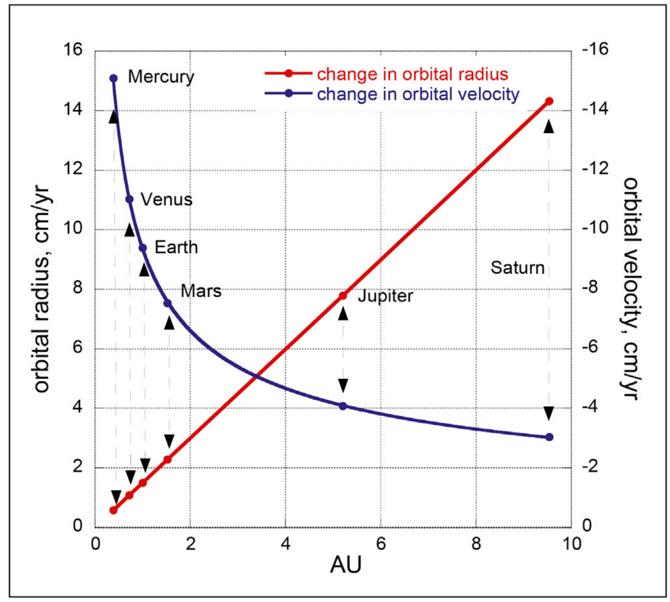

Fig. 1. The effect of a *GM*-dot of $1 \times 10^{-13}\ \mathrm{y}^{-1}$ on the scale and velocity of planetary orbits. The change in orbital velocity is more significant than the change in scale for terrestrial planets. The linear scaling of the solar system is $\sim 1.5\ \mathrm{cm\,y}^{-1}\ \mathrm{AU}^{-1}$.

vectors of the spacecraft and planet, and the planet's *GM* were low (Genova et al., 2017) and did not adversely affect the recovery of *GM-dot*. Almost no other perturbing force increases the along-track position quadratically in time, indicating that the longer the data span the more separable a change in *GM* will be from other perturbing effects.

The accuracy can be substantially better if several baselines are measured over the same time span, particularly if the lines form a closed geometric shape, such as a triangle, providing angle information as well as range and not just as a result of the increase in the number of lines. A closed network of lines provides a geometric constraint on the positions of the planets in at least 2 component directions such that any disturbing influence on one planet will effect the positions of the other planets through the observations. Eventually, the observational constraints will enable significant improvements in the positions of the planets, not just their geometrical relationships, remove ambiguities and weaknesses in the geometry, thereby improving the estimation of other parameters and forces affecting their motion. This is the primary measurement of the Trilogy concept.

In the following sections, we describe a conceptual planetary mission (Zuber et al., 2017) that we anticipate will be able to measure the expansion of the heliocentric orbits of 3 inner solar system bodies, Venus, Earth, and Mar.

4. The Trilogy concept

Trilogy is our concept for an interplanetary ranging constellation of geodetic satellites. Spacecraft are placed in stable orbits around Mars, Venus, and Earth's Moon (or around Earth). We suggest these planetary bodies for several reasons, including the unique signature of planetary orbit expansion happens more rapidly for the inner planets, that they can be reached with modest cruise times, and because ranging to the gas giants over much longer distances will require more capable instrumentation and more stringent pointing requirements than have been demonstrated in space. That said, we recognize the inclusion of other planets would clearly strengthen the overall science result. Once a 3-planet mission demonstrates the value of these inter-planet ranges, the addition of terminals at other planets would be relatively straightforward to implement, either as standalone spacecraft benefiting from rideshare