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Quantum Generalized Euler Heat Equation

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Abstract

Based on nuclear algebra of operators acting on spaces of entire functions with θ-exponential growth of minimal type, we introduce the quantum generalized Fourier-Gauss transform, the quantum second quantization as well as the quantum generalized Euler operator of which the quantum differential second quantization and the quantum generalized Gross Laplacian are particular examples. Important relation between the quantum generalized Fourier-Gauss transform, the quantum second quantization and the quantum convolution operator is given. Then, using this relation and under some conditions, we investigate the solution of a initial-value problem associated to the quantum generalized Euler operator. More precisely, we show that the aforementioned solution is the composition of a quantum second quantization and a quantum convolution operator.

Keywords: Quantum Generalized Euler Heat Equation, quantum convolution operator, quantum generalized Fourier-Gauss transform, quantum generalized Euler operator.
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1. Introduction

As infinite dimensional analogue of the so-called *Euler operator* which is defined as the first order differential operator $\sum_{i=1}^{d} x_i \frac{\partial}{\partial x_i}$ on $\mathbb{R}^d$ (see Ref. [11]), the operator $\Delta_E := \Delta_G + N$ was studied ([8, 9, 26]) based on the white noise analysis, where Δ_G is the Gross Laplacian introduced by Gross [12] and N is the number operator studied in [25] by Piech on infinite dimensional abstract Wiener space as infinite dimensional analogue of the finite dimensional Laplacian. Later on, for $K \in \mathcal{L}(X_{\mathbb{C}}, X'_{\mathbb{C}})$ and $B \in \mathcal{L}(X_{\mathbb{C}}, X_{\mathbb{C}})$ where $X_{\mathbb{C}}$ is the complexification of some real nuclear space X, the operator

$$\Delta_E(K, B) = \Delta_G(\frac{1}{2}K) + N(B)$$

was introduced (see [5]) which is called the (infinite dimensional) *generalized Euler operator*. Moreover it was shown that, under some conditions, $\Delta_E(K, B)$ is the generator of a one-parameter group transformation using the $\mathcal{G}_{K,B}$-transform studied in [16, 8] as well as the existence of a solution of the following *generalized Euler heat equation* was

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