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Dynamic analysis of large structures with uncertain parameters based on coupling component mode synthesis and perturbation method

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Abstract This paper presents a methodological approach to compute the stochastic eigenmodes of large FE models with parameter uncertainties based on coupling of second order perturbation method and component mode synthesis methods. Various component mode synthesis methods are used to optimally reduce the size of the model. The statistical first two moments of dynamic response of the reduced system are obtained by the second order perturbation method. Numerical results illustrating the accuracy and efficiency of the proposed coupled methodological procedures for large FE models with uncertain parameters are presented.

1. Introduction

Dynamic analyses of complex industrial structures by finite element method lead to large finite element models. As the reduction of the order of the model, the system can be condensed by component mode synthesis. Component mode synthesis (CMS) consists in performing the dynamics analysis of structures by a decomposition of the structure into substructures, and these substructures are separately condensed and then coupled. Substructuring techniques differ from the chosen Ritz representation basis for substructure motion; the latter include the vibration normal modes, the rigid body modes, the static modes, the attachment modes, etc. They also differ in terms of the assembling procedures, by elimination or by transformation. Craig and Bampton method [1] uses a basic of fixed-interface eigenmodes and constrained modes; assembly is performed on the junction degree of freedom. The free interface method uses a basic of free-interface eigenmodes and attachment modes. MacNeal [2] includes the static effects of higher normal modes not retained in the component

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representation. Rubin [3] extended MacNeal's method to include the inertial effects of higher normal modes by using a second-order Maclaurin-series, both methods are based on junction force assembling. The junction dofs (jdofs) are therefore missing in the final condensed problem. A method proposed by Bouhaddi and Lambard [4] allows assembling the substructures with the junction degree of freedom.

In most of the CMS method, the resulting condensed problem is conditioned by the number of junction dof. In some cases, the size of coupled system is still large due to the great number of the degree of freedom at the interface. Further reduction of these dofs must often be considered. Bourquin [5] has proposed a method based on the use of the interface modes for Craig–Bampton method and Tran [6] extended this method to various free and hybrid method.

Recently Weng et al. [7] have proposed a substructuring method to calculate the eigensolutions and eigensensitivities for the model updating purposes.

CMS methods are commonly accomplished assuming deterministic behavior of loads and model parameters. However, in many cases the uncertainties associated with model parameters such as geometry, material properties, constitutive law, boundary conditions, and excitation, have to be considered giving arise to stochastic structures.

The analysis of dynamic response of stochastic FE system can be done in the frequency domain using the eigenmodes and frequency transfer functions or in the time domain by a direct integration of the equations of motion, using numerical procedures [8]. The analysis of these stochastic structures commonly seeks the first two moments of the response once the first two moments of the random fields modeling the structural uncertainties are known.

A direct simulation of Monte Carlo [9] is often used and considered as a reference for calculations. Nevertheless, it is in general quite inefficient due to much large number of samples required to guarantee accurate statistical results.

An alternative approach is based on the expansion of the response in terms of a series of polynomials that are orthogonal with respect to mean value operations [10,11]. More precisely, the Karhunen–Loeve expansion is used to discretize the stochastic variables into a denumerable set of random variables, thus providing a denumerable function space in which the problem is cast. The polynomial chaos expansion is then used to represent the solution in this space and the expansion coefficients are evaluated via a Galerkin procedure in the Hilbert space of random variables.

Recent review papers by Stefanou [12] and by Schueller and Pradlwarter [13] summarized the assessment of the past and current status of the procedure for stochastic structural analysis.

The perturbation method based on the Taylor series developments of the response around the average values of the random variables was initiated by Hien and Kleiber [8] to calculate the first two moments of eigenmodes. An improved perturbation method, proposed by Muscolino et al. [14], takes into account the mean and correlation information on uncertain parameters to analyze the dynamic response of structures with mechanical uncertainties under deterministic input. Although, in perturbation method, the variables must have a weak dispersion.

CMS methods are commonly accomplished assuming deterministic behavior of loads and model parameters. Perturbation methods and CMS are used by Hinke et al. [15] to replace numerically expensive operations, such as solving an eigenvalue problem. Sarsri et al. [16] used the CMS coupled with polynomial chaos expansions at first and second orders to compute the frequency transfer functions of stochastic structures.

In this paper various methods of component mode synthesis to reduce the dimensions of the model are used. The first two moments of eigenmodes of structure using a perturbation method are computed. For the needed derivative of various condensed matrices, assembly by transformation is used.

This paper is organized as follows: in Section 2, used CMS methods with fixed and free interfaces method are presented. A procedure of reduction of degree of freedom at the interface in fixed interface and free interface methods is described in Section 3. Various methods of component mode synthesis are used to calculate the first two moments of stochastic eigenvalues and eigenvectors using second order perturbation method in Section 4. Numerical examples are presented to illustrate the efficiency for the proposed technique as well as its accuracy over the whole structure.

2. Component mode synthesis

2.1. Reduced equation of motion

Component mode synthesis (CMS) techniques are well used for static and dynamic in the analysis of large and complex structures. CMS techniques have an advantage of enhancing computational efficiency by reducing the number of degrees of freedom of a structure. An overview of the used CMS is given bellow.

Let us consider a structure, which is decomposed into n_s substructures $\mathrm{SS}^{(k)}$ $(k = 1, \ldots, n_s)$ which do not overlap. For each substructure k the displacement vector $\mathbf{y}^{(k)}$ is partitioned into a vector $\mathbf{y}_j^{(k)}$, called interface dof and $\mathbf{y}_i^{(k)}$ which is the vector of internal dof. The force vector $\mathbf{f}^{(k)}$ is composed into vectors $\mathbf{f}_j^{(k)}$ and $\mathbf{f}_e^{(k)}$, called interface force and external applied force.

In the component mode synthesis methods, the physical displacements of the substructure $\mathrm{SS}^{(k)}$ are expressed as a linear combination of the substructure modes. After some algebraic transformations, a set of Ritz vectors $\mathbf{Q}$ is obtained and the displacements of $\mathrm{SS}^{(k)}$ are expressed as [6]:

$$\mathbf{y}^{(k)} = \mathbf{Q}^{(k)} \begin{Bmatrix} \mathbf{y}_j^{(k)} \\ \boldsymbol{\mu}^{(k)} \end{Bmatrix} = \mathbf{Q}^{(k)} \boldsymbol{\eta}^{(k)} \tag{1}$$

where $\boldsymbol{\mu}^{(k)}$ are the generalized coordinates. Details about the used component mode synthesis methods and related matrices $\mathbf{Q}$ are given in [10] and summarized in Appendix A.

Using Eq. (1) the kinetic energy and the strain energy of each substructure become

$$\begin{aligned} T^{(k)} &= \frac{1}{2}\, {}^T\dot{\boldsymbol{\eta}}^{(k)} \mathbf{M}_c^{(k)} \dot{\boldsymbol{\eta}}^{(k)} \\ U^{(k)} &= \frac{1}{2}\, {}^T\boldsymbol{\eta}^{(k)} \mathbf{K}_c^{(k)} \boldsymbol{\eta}^{(k)} \end{aligned} \tag{2}$$

where $\mathbf{M}_c^{(k)}$ and $\mathbf{K}_c^{(k)}$ are the condensed matrices of the substructure (k) given by